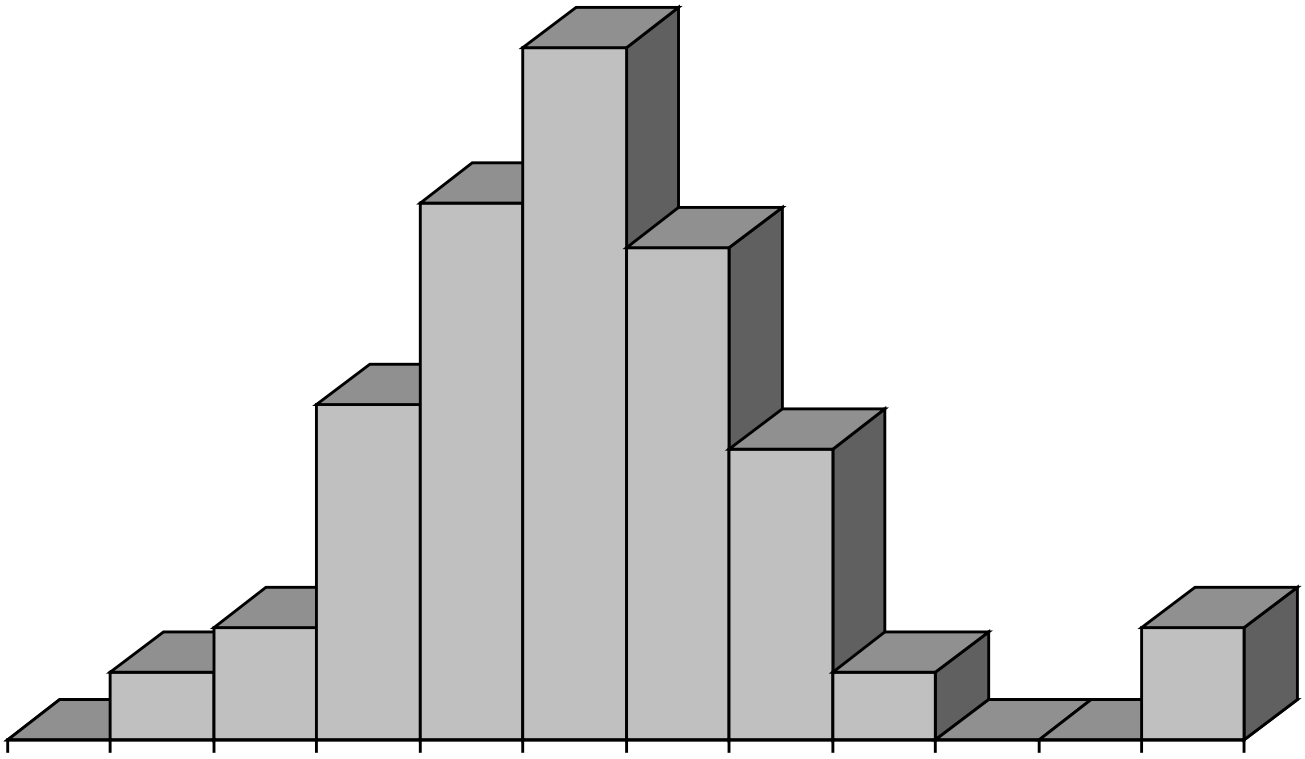
Idaho
Ratio Study Manual



2023 - 2024

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TITLE PAGE

The 2023 - 2024 IDAHO RATIO STUDY MANUAL has been prepared by Alan Dornfest, Property Tax Policy Bureau Chief.

This manual supersedes any previous ratio study manual and is in effect beginning January 1, 2024.

2023 - 2024 IDAHO RATIO STUDY MANUAL

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INTRODUCTION

INTRODUCTION TO RATIO STUDY PROGRAM

Annual studies of the ratio between the market value determined by verified sales prices of real property and the assessed value of the same real property as stated on county assessors' rolls are conducted. These studies provide technical assistance to counties, test the results of the continuing appraisal process, and assist the State Tax Commission in its task of equalizing and certifying county and railroad property values. The ratio study is also used to certify adjusted market value for the Boise School District and affects the amount of property tax that district may levy.

This manual explains procedures involved in ratio studies. Areas discussed include the following:

1. Sampling procedure, including sales verification process;
2. Types of studies;
3. Statistical analysis of data;
4. Education program and technical assistance;
5. The ratio study as an appraisal tool;
6. Standards and equalization procedures;
7. Definitions.

Examples are shown to help clarify the statistics presented and there is a section demonstrating how the ratio study can be used in the appraisal process. Historical background is also presented. The current manual adopts standard IAAO terminology whenever possible.

The reader will find general information as well as complex formulas. It is far more important to understand the concepts presented so that the ratio study can be used in mass appraisal work.

Ratio Study System in Place 2007 - 2024

Beginning with the 2007 ratio study, barring county board of equalization action that necessitates more comprehensive study, only primary categories as defined in Rule 130 have been tested with ratio studies. Secondary categories are also tested whenever county board of equalization action changes the compliance status of primary categories previously found to be in compliance. Rule 131 governs this process.

Except for the ratio studies conducted for the Boise School District and for the equalization of railroad property with commercial and industrial property, the median and the median confidence interval will continue to be used to test compliance with ratio study standards. For county ratio studies, the system presented in Rule

131 is continued and includes the following:

- Categories to be tested will include the following:
  - Improved Residential (including manufactured homes on same ownership land);
  - Unimproved Residential;
  - Improved Commercial;
  - Unimproved Commercial;
  - Manufactured homes on leased land.
- If equalization adjustments are warranted, such adjustments would be applied to any component category for which at least one observation is included in the sample (*see Rule 131 for exceptions*).
- 90% (two-tailed) confidence intervals will still be used initially to test compliance. We will continue to test compliance with lower (80%) confidence intervals around the median and may base equalization decisions on sample median when 80% intervals fail to be not within the 90% - 110% range for two consecutive ratio studies.
- Although not subject to formal ratio studies, agricultural land categories will be subject to preliminary and final studies of assessment level and held to standards as described in Rules 130 and 131.

Boise School District Ratio Studies:

- The weighted mean ratio will be used, except when distorted by non-representative ratios. In this instance, the median will be used.
- Equalization adjustments will only be considered when the appropriate (weighted mean or median) confidence interval fails to include 95% or 105%.

Additional information concerning this procedure is found in rules 130 and 131, found in Appendix VII, and in the "Standards and Equalization Procedures" section of this manual.

Railroad Equalization Studies:

In addition to county by county analysis of the level of assessment of commercial and industrial property, since 2015 there have been special studies to ensure that commercial and industrial property has an assessment level no lower than 95% within the area of the state to which any given railroad has value apportioned. To determine compliance, special commercial and industrial ratio studies are

conducted by combining all such valid sales in counties to which a given railroad's value will be apportioned. Compliance is based on the upper 90% confidence interval limit around the weighted mean. This limit must be no lower than 95%. If the result shows non-compliance for any railroad, the Idaho value of the railroad will be adjusted downwards based on the difference between the sample weighted mean ratio and 100%.

In 2023, the above procedure was extended and was applied to all operating property companies. In 2024, the procedure was further modified to be based on commercial and industrial property ratio studies using 2024 assessed values and sales time adjusted to January 1, 2024.

Manual vs. Rules:

This manual is intended to provide information for training and technical assistance. Compliance standards stated in this manual are advisory in nature, unless specifically incorporated into State Tax Commission rules. Statutorily set ratio study guidelines for school district ratio studies can be found in Idaho Code §63-315. The requirement for equalization of categories of property by the State Tax Commission is found in Idaho Code §63-109. Assessment level compliance standards are set by State Tax Commission rule 131, while school ratio study procedures are described in rule 315. Federal law, known as the "4Rs Act," includes requirements for commercial and industrial property to be assessed no more than 5% lower than railroad property.

HISTORICAL BACKGROUND

To better understand the ratio study, it is important to understand some of the history of the assessment process in Idaho and how equalization has fit into this process. This section is intended as an undocumented outline of the past 50 years, designed to give a broad, general overview of this period.

Evolution of the Valuation Process

In reviewing the valuation process over time, several distinct periods appear to exist. These can be summarized as follows:

Pre - 1965:	Counties establish different assessment ratios (levels) independently and without state direction.
1965:	The legislature mandates ratios of 20% for locally assessed real and personal property and 40% for centrally assessed operating property.
1967:	State Supreme Court rules classification (see 1965 case) unconstitutional and requires all property to be assessed at 20%. This level is to be phased in by 1979 (later revised to 1982).
1967-1978:	Declared ratios of each county approach 20%; actual ratios lag further behind each year, with the lowest ratios found in residential categories.
1978-1980:	1% Initiative passes with requirement that property be assessed at full market value as of December 31, 1978; the reassessment is to be completed in time for the 1980 rolls. This results in a typical residential level of about 80% of market value.
1981-1982:	For 1981, the December 1978 values are to be increased by 4.04% (2% for 1979 and the same for 1980). Full current market value is to be achieved for 1982.
1982-1989:	Through 1987, current market value is established each year, based on sales centering one year prior to the lien date. Beginning in

1988 sales data centers 6 months prior to the lien date (prior calendar year).

1989-1991: Current market value each year is still required. Prior calendar year sales are used to test and provide information for ongoing appraisals.

1992-present: Current market value is still required, with prior year sales used for ongoing (county) appraisals, but the last three months of the prior year and the first nine months of the current year are used in most ratio studies to test assessment conditions.

Evolution of the Ratio Study and Equalization Program

The ratio study and its use changed and grew during this same period. Changes can be outlined as follows:

Pre - 1960: There were sporadic studies, with a partial study in 1955 and a full study of each county in 1958. Use is unknown.

Mid 60's - 1978: Annual studies used for school equalization purposes. Equalization based on county-wide weighted average assessment level, restricted to school funds only; did not equalize inequity between categories.

1979 - 1981: Transition to current system; no equalization.

1982 - 1987: State ordered trending by category if out of compliance with level standards.

1988 - 1991: State ordered trending only if category out of compliance for two successive ratio studies.

1992 - 1994: Compliance determined with burden of proof of non-compliance on State Tax Commission. Two successive non-compliance studies required to produce a trending order.
School equalization reinstated beginning with 1993 ratio study.

1995 - present:	Equalization orders may result after just one year of non-compliance. Special follow-up studies test current (not just past) year's assessment conditions prior to final compliance determination. Beginning with 2002 ratio study, burden of proof of non-compliance lowered given two consecutive years with sample measures of level outside acceptable range.
2015 - present:	Equalization extended to railroad values.
2019 – present:	Equalization extended to farmland.
2023 – present:	Equalization extended to all operating property.

Note: School district ratio studies were discontinued in 2006, except for the Boise School District.

RATIO STUDY SAMPLING PROCEDURE

For the most meaningful ratio study, sales information must be collected, confirmed, and verified to prepare samples of arm's length, market value transactions which are representative of each category or type of property to be studied. The ideal sample would be:

1. Randomly selected;
2. Proportionally representative of all locational influences and pertinent property characteristics;
3. Of sufficient size to be considered reliable.

Since there presumably is not an equal opportunity for each property to sell, provide information, and be in the ratio study, the randomness test is not met. Similarly, requirements 2 and 3 are somewhat uncontrollable. Therefore, to maintain some degree of statistical validity, sample size goals should be as follows:

1. Obtain the greatest possible number of acceptable sales;
2. Do not exclude any sales unless verifiably invalid or if these sales over-represent certain properties;
3. Make additional efforts to obtain sales in areas and categories which, traditionally, have few verified transactions;
4. Check for over-representation of "hot spots". Do not allow these areas to contain more sales than the proportion of property in the "hot spot" to the category being tested. Note that the "hot spot" designation also applies to over-representation of certain value related property characteristics. For example, if 20% of the residential improvements have more than 2500 square feet of living area, and this group accounts for 50% of the ratio study sample, sales should be removed randomly until the correct proportions are achieved.

Because of our inability to randomly sample property, we cannot truly estimate the number of sales necessary to produce a reliable and valid ratio study. However, it should be noted that the major factors which influence sample size requirements are:

1. Uniformity: Fewer sales are needed to study areas with good assessment uniformity.

2. Acceptable error: If a larger error in results is considered acceptable, a smaller sample size is indicated.

The number of parcels in an area or category has only a minor influence on sample size. The following examples use standard statistical sample size formulas to demonstrate these influences:

	Example A	**Example B**	**Example C**
Total # of parcels	5,000	5,000	500
Standard deviation: (low numbers = good)	15%	25%	25%
Acceptable error	± 10%	± 5%	± 5%
Required sample size:	9	98	84

The procedure used to derive sample size in these examples assumes that a random sample of any size can be produced. Because this is not true in ratio study sampling, sample size formulas are of only marginal use in establishing ratio study sample sizes.

The primary source of sales information will be the deed records of the county. All open market sales which are not to be excluded as invalid, as indicated below, should be included in the ratio study, unless such inclusion can be demonstrated to produce over-representation of certain value influences. Validity of sales data should be determined by confirming the details of each transaction.

Confirmation may be made by contact, in person or by mail, with either the grantee, the grantor, or other knowledgeable person who is fully informed of the terms of the transaction. Sales may also be confirmed by review of sales documents. These documents include:

1. Purchase agreements,
2. Escrow documents, and
3. Broker records.

When any portion of the property studied is exempt from property taxes, the sale must be adjusted. For homeowner's and hardship exemptions, the exempted value should be added back to the taxable value of the property before the ratio is calculated. For all other exemptions, each sale price should be adjusted to remove the exempted value before the ratio is calculated. If the adjustment cannot be calculated, the sale should be deleted from the study.

A sample which includes personal property may be used in the ratio study if the selling price can be adjusted satisfactorily to eliminate the personal property value. When such adjustments are considered, the amount to be subtracted from the sale price should be market derived and should not merely be the cost of the personal property.

The following sales situations are presented to illustrate the types of transactions usually presumed to be unsuitable for use in an assessment ratio study:

1. The deed does not show warranty of title by the grantor; however, other types of deeds may be used in the ratio study if verification proves they are bona fide transactions.

2. The subject of the grant is a partial interest.

3. The grantee or grantor is a federal, state, county, municipality or other political subdivision, or is a public utility.

4. The grantee is a bank, insurance company, building and loan association, or other financial institution.

5. The grantee is a charitable, educational, or religious institution.

6. The grantee and grantor are related by blood or marriage, or are corporate affiliates. Sales between business associates should be carefully screened.

7. The grantee and grantor are the same and the deed is a convenience to change the nature of the interest in the property. (Example: Tenancy in common to tenancy by the entireties.)

8. The subject property constitutes or is a part of a trade or exchange of properties.

9. The grantor is transferring property to avoid a lien or judgment.

10. The sale results from judicial order, decree, or proceedings, and grantor is a sheriff, receiver, or other court officer.

Sales that can be clearly identified as falling into any of the above categories are considered potential candidates for rejection from the ratio study.

It is important to note that sales between relatives should not automatically be excluded. Often, sales prices are not demonstrably influenced by family relationships and this effect should be determined when verifying sales. IAAO guidelines and rule 131 regarding foreclosure related sales indicate that these sales may influence the broader market and should therefore not be automatically excluded. Inclusion is especially important when such

sales become dominant in an area. This is deemed to be the case provided such sales comprise more than 20% of the sales available in any primary category.

The ratio that is calculated for any sale should not be used as an indication of validity. However, studies have shown that extreme outlier ratios (very high or very low) often indicate doubtful sample validity. Since outliers can substantially impact measures of assessment level in small samples, non-typical ratios should be used as flags to identify sales which should then be subject to additional verification. As a rough rule of thumb, sales with ratios outside of a range of ±2 standard deviations around the mean should be reviewed. However, there are no automatic, percentage based exclusion procedures. Alternate procedures to be employed are based on the inter-quartile range and can be found in the IAAO 2013 *Standard on Ratio Studies*.

Example 1: Outlier Review Guide

Sale #	Assessed Value	Sales Price	Ratio (%)
1	15,000	25,000	60.00
2	15,000	22,000	68.18
3	17,000	20,000	85.00
4	19,000	22,000	86.36
5	25,000	27,000	92.59
6	24,000	25,500	94.12
7	25,000	25,000	100.00
8	20,000	16,000	125.00
9	35,000	25,000	140.00
10	55,000	25,000	220.00

Although the assessment level appears to be acceptable in this example, the mean is 107.13% and uniformity shown is very poor (COD = 30.74%). The standard deviation is 46.22%. Sale #10 exceeds the mean by more than 2 standard deviations and should be reviewed. If sale #10 were found to be invalid, the mean would become 94.6% and the standard deviation 25.2%.

The procedure for rejecting sales is:

1. Sales to be included are submitted by the county assessor to the consulting appraiser. The consulting appraiser may determine that additional sales are needed and may search for and include these when possible.

2. The consulting appraiser and the county assessor should review the sales and the consulting appraiser may use discretion to delete invalid sales found in the study. If there is disagreement between the consulting appraiser and the assessor, the assessor should make a written recommendation to the State Tax Commission regarding sales to be added or eliminated.

3. The State Tax Commission decides whether to follow the county's recommendations and notifies the county and consulting appraiser accordingly.

Contracts for sale are usable in the ratio study if the conditions of the sale meet the requirements of a bona fide, arm's length transaction. Implicit in this term, arm's length transaction, is the consummation of a sale as of a specified date and the passing of title from seller to buyer under conditions whereby:

1. buyer and seller are typically motivated.
2. both parties are well informed or well advised, and each acts in what is consider their own best interest.
3. a reasonable time is allowed for exposure in the open market.
4. payment is made in cash, or with financing which is on terms generally available in the community at the specified date and typical for the property type in its locale.
5. the price represents a normal consideration for the property sold and is unaffected by special financing amounts and/or terms, services, fees, costs, or credits incurred in the transaction.

RULES

The basic guidelines and standards for the ratio study are contained in the following State Tax Commission rules, of which 130, 131, and 315 are included in this manual:

Rule 217:	Rule pertaining to market value and appraisal.
Rule 130:	Rule listing and describing primary property categories.
Rule 131:	Rule pertaining to use of ratio study in equalization.
Rule 315:	Rule pertaining to Boise School District ratio studies.

TYPES OF STUDIES

The type of ratio study varies with intended use. Studies may be used for many purposes, including:

1.) determining assessment conditions for property of a particular type or class, or at a particular location;

2.) establishing baseline conditions prior to reappraisal and monitoring the progress of reappraisal work;

3.) equalizing property values to ensure equal treatment by category and equal effect of exemptions;

4.) computing adjusted market value for school equalization and levy purposes;

5.) computing adjustments for railroad values to ensure compliance with federal law and eliminate ratio discrimination between commercial and industrial property and railroads.

Depending on intended use, the time period from which sales to be included are to be drawn will vary, and the date of assessments against which these sales are to be compared will also vary.

Studies by Counties

Studies done by local officials will generally relate to purposes (1) and (2) shown above. These studies should, ordinarily, involve sales occurring during the calendar year immediately preceding the assessment year. For example, for year 2024 assessments, calendar year 2023 sales should be used. However, this should not be considered to be inflexible. If sufficient sales are not available during one year, it is permissible to extend the sales period, provided that proper, documented time adjustments are developed and that economic conditions have not become greatly altered.

Sales data for 2023 (or a longer period, if necessary) could be compared to either 2023 or 2024 assessments, depending on whether the study was designed to determine initial (baseline) or final assessment conditions for 2024. Comparison to 2023 assessments would also represent a final review of those assessments, and would help the assessor to determine the accuracy and validity of decisions made and data used for that year.

For every period of sales used in a ratio study, time adjustments must be considered. When using the prior calendar year's sales, the

sale prices typically will reflect market values as of July 1 of the prior year. Since the assessment date is six months later, sale prices should be time adjusted forward to reflect value as of January 1 of the assessment year. Procedures to use to determine appropriate time adjustments are found in the IAAO Property Appraisal and Assessment Administration (PAAA, IAAO 1990), Mass Appraisal of Real Property (IAAO, 1999), and Fundamentals of Mass Appraisal (IAAO, 2011) textbooks.

Special studies reflecting various geo-economic areas, and classes or types of properties are strongly recommended. Although the county is required to appraise all property at least once every five years, special emphasis should be focused on those areas which have poor uniformity as demonstrated by ratio studies.

County Equalization Ratio Study

Each category of property must be in compliance with assessment level standards each year. County equalization ratio studies test compliance and are authorized under Idaho Code §63-109 and State Tax Commission rule 131. Although the principles apply to each year's ratio study, this manual illustrates equalization ratio studies using the 2023 final county ratio study as an example. Typically, the 2023 final county ratio study is by primary category of property based on sales and assessments as follows:

> Sales occurring between Oct. 1, 2022 and Sept. 30, 2023 are adjusted for time (to January 1, 2023) and compared to 2023 assessments. This study is completed in March, 2024 and is considered a final report on 2023 assessment conditions.
>
> Time adjustments must be considered and made whenever provable in the market. Different adjustments may be necessary to reflect different amounts of appreciation in different categories of property.
>
> Counties will be notified of any category that is out of compliance (see: "Standards and Equalization Procedures" section). Results that are out of compliance trigger a follow-up study which will compare 2024 assessments with sales occurring between January 1, 2023 and December 31, 2023. For this follow-up study, sale prices will be time-adjusted to January 1, 2024.
>
> Any category that is not considered in compliance after completion of the follow-up study will be reported to the county board of equalization for corrective action. If such action is not taken or fails to restore a category to compliance, the results will be

reported to the State Tax Commission, which may take equalization action at its August, 2024 meeting. The State Tax Commission may delay implementation of any equalization adjustment for one year, if there is reason to question the representativeness of the ratio study. The State Tax Commission may also expand the sales time period when it is necessary to do so to obtain representative samples. The Commission may also have consulting appraisers conduct appraisals and these may be included with sales samples to improve sample size and representativeness.

Statistically, the burden of proof of noncompliance is on the State Tax Commission as the equalizing agency. Typically, a conclusion of county non-compliance will be reached when the State Tax Commission is at least 95% certain that the median level of assessment is **not** between 90% and 110% of market value for a given category of property. However, lower degrees of certainty can be used when point estimates continue to fall outside the acceptable range for multiple years (see: "Standards and Equalization Procedures" section and Rule 131 for more complete explanations).

Perspective on the Ratio Study & Equalization

The use of the ratio study as outlined in this section conforms with major features of the IAAO *2013 Standard on Ratio Studies* (although no provision in the Idaho system tests for differences between categories of property in a county - the IAAO *Standard...* suggests no more than a 5% permissible difference). This *Standard...* advocates use of sales spanning or after the assessment date for equalization purposes. The use of sales following the assessment date creates an independent check of assessed values and lessens the need for additional monitoring (checking assessment rolls, etc.) to confirm that ratio study results are representative of selling and non selling parcels. The equalization ratio study becomes simply an audit procedure to determine compliance and the need for equalization adjustments. This particular study is not designed to assist in the appraisal process or to otherwise provide technical assistance to the county. Those functions are met by ratio studies done locally or with State Tax Commission assistance, based on sales from an earlier time frame.

Ratio study sampling procedures rely on sales which may not occur in random patterns and, thereby, may not conform with standardized statistical survey sampling procedures which enable precise calculation of reliability. For any statistical validity in both equalization and reappraisal ratio studies, sample representativeness is critical. If, for example, a new area begins to sell after the assessment date, an influx of sales from this area may cause over-

representation of assessment conditions which differ significantly from those in the remainder of the category. Accordingly, samples may be adjusted through additions or deletions to improve representativeness. Follow-up studies on non-complying categories of property will aid in ascertaining assessment conditions and will more nearly reflect value changes made by assessors for the current year subject to equalization.

Limitations

The most difficult areas to appraise or equalize are those with highly erratic markets or with few sales. The current ratio study standards greatly reduce the possibility of equalization adjustments in categories with poor reliability due to small samples or poor uniformity, because the burden of proof of noncompliance is on the State Tax Commission and because secondary categories usually are studied in combination, rather than separately. Under current standards, even in the largest county or category, every category with a median between 90% and 110% of market value is considered in compliance. Categories represented by small, poorly reliable samples are allowed considerable deviation from this range, as expressed by the sample median (known in statistics as a point estimate). For example, in the first year of testing, a sample of 10 sales with median of 85% and a median 90% confidence interval of 76% - 92% would not be considered out of compliance (on the low side). After two years with an 80% confidence interval would be required to overlap the 90% - 110% range for the category to be considered in compliance. This is more stringent, since 80% confidence intervals are nearly always narrower than 90% confidence intervals. Additional charts and information on compliance are presented in the "Standards and Equalization Procedures" section.

Often, due to limited sales in sparsely populated areas, certain categories of property have assessment conditions determined by analysis of a small number of sales. Rule 131 precludes ratio study based equalization if a minimum of 5 sales (or sales and independent appraisals) is not available. Any analysis of any sample with fewer than five sales is intended only as a guide to the assessor. The State Tax Commission may, at its discretion, add appraisals conducted by Commission staff to small samples to improve representativeness and attain minimum sample size for equalization studies.

The importance of maximizing the amount of sales data to enhance study reliability cannot be over-emphasized.

Ratio Study use in School Equalization

Idaho Code §63-315 imposes a requirement for the State Tax Commission to compute adjusted market value for the Boise School District and to publish the statistical measures computed in the ratio study done to fulfill this requirement.

Procedure

The weighted mean ratio is used for computing the Boise School District's adjusted value unless distortion can be proven, in which case the median is substituted. The compliance range is 95% - 105% and there will no adjustment to category values provided a 90% two-tailed confidence interval around the weighted mean (or median, if appropriate) overlaps this range. This ratio study is conducted by property "designation" instead of property category. All categories are to be assigned to one of two designations:

1. Residential, including manufactured housing or
2. Commercial.

A complete discussion of procedures to be used is found in Rule 315.

Discussion

Actual calculation of the Boise School District's adjusted market values is done using spreadsheet software. Copies of all calculations will be made available electronically on request.

Statistical Measures

Statistical measures are computed using the sales and appraisal samples described earlier in this report. All measures are computed in accordance with standard statistical procedures described in this manual and in the IAAO 2013 *Standard on Ratio Studies.*

STATISTICAL ANALYSIS OF DATA

In conducting assessment ratio studies the State Tax Commission performs statistical analysis of all verified sales and appraisals to be used. The results of this analysis are compiled and presented on tables, which identify the area being studied and category(ies) included (see Appendix I).

Results shown include statistical measures of central tendency (level), variability (uniformity), and reliability (precision). Tests of assessment progressivity/regressivity and of the normality of the distribution of assessment ratios are included whenever possible. Worksheets demonstrating procedures for many of these statistical tests can be found in Appendix III.

MEASURING ASSESSMENT LEVEL

Measurements which determine assessment level do so by establishing what is known statistically as the central tendency of the observations (in this case, the ratios). The goal is to determine one number which best represents assessment level. The number is based on the available sales data and is computed from ratios found by dividing each assessed value by the sale price of that property.

Assessment ratios can be expressed in decimal or percent form and are calculated in the following way:

$$\text{ratio} = \frac{\text{Assessed Value}}{\text{Sales Price}}$$

The term, **A/S** also means "ratio".

Example 2:

Assessed Value = $40,000
Sales Price = $50,000

$$\text{ratio} = \frac{\$40,000}{\$50,000} = 0.80 = 80\% = \text{A/S}$$

Using a ratio of 100% as a proxy for market value and as the primary point of reference, ratios will always fall into 3 groups:

(perfect) A. ratio = 100%; Assessed Value = Sales Price

(low) B. ratio < 100%; Assessed Value < Sales Price
(< means less than)

(high) C. ratio > 100%; Assessed Value > Sales Price
(> means greater than)

Although the ideal ratio is 100%, in practice it is rarely possible to precisely predict the selling price of individual properties. This prediction is further complicated by the fact that most properties are not currently for sale.

In mass appraisal aiming at market value, we expect approximately equal numbers of properties to sell for more than, or less than, their assessed values. Appraisal errors or marketplace uncertainty should be random. In other words, if the goal is market value (100%), individual properties randomly should be expected to appear to be assessed too high or too low, but a category as a whole, measured by a sufficient number of representative sales, should appear to be assessed close to 100%.

In Idaho, we calculate four different tests of assessment level. The purpose of each test is to discover whether differences between assessments and sale prices are random, individual events, or are systematic, resulting in low or high overall level of assessment. Four tests are used because each test is subject to different types of distortion or bias, the effect of which is minimized by reviewing the four results. The tests and their identifying symbols are:

1. Mean $\left(\overline{A/S}\right)$, also known as:

    a. Arithmetic Mean or simple Average

    b. Unweighted Mean or Unweighted Average

2. Median $(\widetilde{A/S})$

3. Geometric Mean

4. Weighted Mean $(\overline{A}/\overline{S})$, also known as: Sales Weighted Mean

Each of these measures is calculated on the sales or sales and appraisal ratios that constitute the sample. The results are point estimates or statistics related only to the sample. Additional tests of the reliability of these statistics are necessary to draw inferences about the population of unsold and sold properties that the sample is designed to represent (see "Statistical Measures of Reliability" section).

Mean $\left(\overline{A/S}\right)$:

The mean ratio is determined by summing the ratios computed for each sale and dividing this total by the number of sales in the sample. (See computation in Example 6 following.)

The mean has the advantages of being simple to compute and easy to understand. Another advantage is that it takes all measurements into account. It is also used as a basis for certain measurements of uniformity. However, a small number of very low or high assessment ratios tend to weigh heavily on the mean, distorting it (usually on the high side) so that it is often not the truest measure of assessment level. In fact, because of the mathematics involved in ratios, the mean is biased on the high side, although this is not always apparent. In small samples this distortion tends to be more pronounced. There is also a tendency to overuse this statistic. It is important to realize that a mean of 100% does not necessarily mean good assessment conditions.

Geometric Mean:

The geometric mean is a measure of assessment level which is not as susceptible to influence from a few extremely high ratios as the arithmetic mean. It does not, however, correct for problems caused by low ratios and will never be higher than the arithmetic mean. The geometric mean also suffers from being more complex and therefore less understandable. Finally, there are no corresponding measures of reliability to test the precision of this statistic. Example 6 shows the geometric mean for a sample and presents a comparison to the mean.

Geometric Mean:

A measure of level determined by multiplying all of the ratios in a sample together and then taking the "n"th root of the product of this calculation.

geo. mean = $(A_1/S_1 * A_2/S_2 * A_3/S_3 *... A_n/S_n)^{1/n}$

where A_n/S_n represents each ratio in the sample;
and n = the number of ratios in the sample.

Median $\widetilde{(A/S)}$:

The median ratio is an indicator of the central ratio in any sample. It is determined by arraying all of the ratios from a particular category and finding the midpoint. Again, it is possible to compare this value to both the arithmetic and geometric means, with large differences indicating problems in either sampling or county appraisals. The median is also used in calculating the coefficient of dispersion, discussed under the uniformity heading. The median is considered an unbiased estimator of level, since it is not subject to the effects of outlying ratios; however, this can be a disadvantage as well as an advantage, since valid outliers are not reflected.

Once the ratios are computed and arrayed, the rank or order number corresponding to the median ratio can be found from this formula:

median rank = .5(n) + .5,
where n = the number of sales in the sample.

Example 3:

Sale #	Ratio
1	80%
2	85%
3	90%
4	95%
5	100%

Median rank = .5(5) + .5 = 3
The third ratio is 90% and this is the median.

Example 4:

Sale #	Ratio
1	80%
2	85%
3	90%
4	95%
5	100%
6	105%

Median rank = .5(6) + .5 = 3.5

In Example 4, the median ratio is between the third and fourth ratios. These two ratios are added together with the sum averaged (divided by 2) to compute the median:

ratio #3 : 90%
ratio #4 : 95%
sum : 185%

sum / 2 : 185% / 2 = 92.5%

The median is 92.5%

(Note: ( ) adjacent to a number indicate that the number is to be multiplied by whatever is inside the ( ).)

Weighted Mean ($\overline{A}/\overline{S}$):

The weighted mean differs from the mean in that the computation is based on the total assessed value for the entire sample divided by total of all sales prices for all sales in the sample. (See computation in Examples 7 and 8 following.)

In the determination of this statistic, sales of more expensive property weigh more heavily and exert more influence on the result than those of less costly property. Outlying individual ratios do not exert strong influence on this statistic, but cautious use is recommended, since value weighting may cause considerable distortion, particularly by very high value occasional sales which may have non-representative low ratios.

Examples comparing the mean, median, and weighted mean follow:

COMPARISON OF THREE MEASURES OF ASSESSMENT LEVEL:

Example 5: WTD. MEAN = MEAN = MEDIAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 20,000	$ 50,000	40.00%
2	30,000	50,000	60.00%
3	40,000	50,000	80.00%
4	50,000	50,000	100.00%
5	60,000	50,000	120.00%
6	70,000	50,000	140.00%
7	80,000	50,000	160.00%
Totals:	**350,000**	**350,000**	**700.00%**

MEAN = 100.00%
MEDIAN = 100.00%
WTD. MEAN = 100.00%

Measures of level are considered biased if they tend to distort the impression of the true assessment level. In Example 5, the three measures are equally useable with distortion caused only by different assessments of properties treated identically in the market.

COMPARISON OF MEASURES OF ASSESSMENT LEVEL:

Example 6: MEAN > MEDIAN & SALES WTD. MEAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 80,000	$ 50,000	160.00%
2	75,000	60,000	125.00%
3	70,000	70,000	100.00%
4	65,000	80,000	81.25%
5	60,000	90,000	66.67%
6	55,000	100,000	55.00%
7	50,000	110,000	45.45%
Totals:	**455,000**	**560,000**	**633.37%**

MEAN = 90.48%
MEDIAN = 81.25%
WTD. MEAN = 81.25%
GEOM. MEAN = 82.98%

COMPUTATIONS:

633.37/7 = 90.48%	(mean) (the "/" means to divide)
.5 x 7 + .5 = 4 = 81.25%	(median)
455,000/560,000 = 81.25%	(wtd. Mean)
160 x 125 x 100 x 81.25 x 66.67 x 55 x 45.45 = $(27081979031250)^{1/7}$ = 82.98%	(geometric mean)

In Example 6, the mean is distorted by high ratios. The best indicator of level is probably the median, with ratios exceeding this point by up to 80 points, while the lowest ratio is within 36 points. Note that in comparison to the mean, the geometric mean is affected to a much lesser extent by the high ratio sales.

COMPARISON OF THREE MEASURES OF ASSESSMENT LEVEL:

Example 7: WTD. MEAN >> MEAN & MEDIAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 10,000	$ 20,000	50.00%
2	20,000	40,000	50.00%
3	30,000	60,000	50.00%
4	60,000	80,000	75.00%
5	90,000	100,000	90.00%
6	120,000	120,000	100.00%
7	160,000	140,000	114.29%
Totals:	**490,000**	**560,000**	**529.29%**

MEAN = 75.61%
MEDIAN = 75.00%
WTD. MEAN = 87.50%

COMPARISON OF THREE MEASURES OF ASSESSMENT LEVEL:

Example 8: WTD. MEAN << MEAN & MEDIAN

Sale #	Assessed Value	Sale Price	Ratio
1	$ 40,000	$ 20,000	200.00%
2	60,000	40,000	150.00%
3	60,000	60,000	100.00%
4	75,000	80,000	93.75%
5	60,000	100,000	60.00%
6	70,000	120,000	58.33%
7	30,000	140,000	21.43%
Totals:	**395,000**	**560,000**	**683.51%**

MEAN = 97.64%
MEDIAN = 93.75%
WTD. MEAN = 70.54%

The weighted mean fails as a valid indicator in Example 7, where high ratios on higher priced property distort this measurement upwards. The opposite occurs in Example 8, where assessments are too high on lower priced property and too low on higher priced property. The weighted mean is questionable in this case as well.

The following chart provides a summary of assessment level statistics:

Assessment Level		
	Advantages	**Disadvantages**
Mean	Uses all data; basis for uniformity and reliability statistics	Biased high (affected more by high ratios)
Weighted Mean	Eliminates distortion due to high or low ratios	Price related weighting distorts toward ratios on higher priced property
Median	Unbiased (by extreme data)	Ignores all but 1 or 2 ratios; limited predictive capability
Geometric Mean	Unbiased (by extreme high ratios)	Not useful as basis for uniformity and reliablility statistics

MEASURING ASSESSMENT UNIFORMITY

Uniformity determines the quality and inherent equity of property assessments. Although both the appraisal and the market transaction are subject to distortion on any individual property, if the magnitude of this distortion is consistently large, taxes paid by similar properties in the same area will differ widely. The goal of a fair assessment program is to reduce inequity of this type.

There are two overall types of inequity that can occur:

1. Inequity between categories.
2. Inequity within a given category.

In the first case, inequity results when the assessment level is lower in one category than another. This situation becomes apparent when level indicators from different categories are compared.

In the second case, the distortion is entirely within one category and is not indicated by measurements of level. The following chart illustrates this situation:

LEVEL VS. UNIFORMITY

Example 9:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 10,000	$ 25,000	40.00%
2	30,000	50,000	60.00%
3	22,500	30,000	75.00%
4	60,000	60,000	100.00%
5	37,500	30,000	125.00%
6	70,000	50,000	140.00%
7	40,000	25,000	160.00%
Totals:	**270,000**	**270,000**	**700.00%**

MEAN = 100.00%

MEDIAN = 100.00%

WTD. MEAN = 100.00%

GEOMETRIC MEAN = 90.68%

* MEASURES OF ASSESSMENT LEVEL

(COD) COEFFICIENT OF DISPERSION = 35.71%

(COV) COEFFICIENT OF VARIATION = 44.06%

* MEASURES OF UNIFORMITY

Although all of the level measurements indicate that market value has been achieved on an overall basis, significant variation is present and will prevent many taxpayers from being taxed equitably.

If the seven sales shown accurately represent assessment conditions, each sale can be thought of as representing about 14% of the property. In other words, 14% of the property is assessed at 40% of value, 14% at 60%, and so on. The type of inequity demonstrated in Example 9 is known as **horizontal** inequity, because it occurs throughout a category of property and is not more pronounced on higher or lower priced property.

LEVEL VS. UNIFORMITY

Example 10:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 21,000	$ 25,000	84.00%
2	44,000	50,000	88.00%
3	28,000	30,000	93.33%
4	60,000	60,000	100.00%
5	32,000	30,000	106.67%
6	56,000	50,000	112.00%
7	29,000	25,000	116.00%
Totals:	**$ 270,000**	**$ 270,000**	**700.00%**

MEAN	=	100.00%	*
			* MEASURES
MEDIAN	=	100.00%	* OF
			* ASSESSMENT
WTD. MEAN	=	100.00%	* LEVEL
			*
GEOMETRIC MEAN	=	99.36%	*

(COD) COEFFICIENT OF DISPERSION	=	9.90%	* MEASURES
			* OF
(COV) COEFFICIENT OF VARIATION	=	12.17%	* UNIFORMITY

In Example 10, the sales with prices identical to those in Example 9 have closer assessed values. Measurements of level are unchanged except for the geometric mean, which is now closer to the other measures. However, variation between ratios of assessment has been reduced and there will be much better taxpayer equity.

Determining Uniformity

Available procedures and statistics which enable uniformity to be calculated or visualized include:

1. Range
2. Frequency Distribution
3. Histogram
4. Coefficient of Dispersion (COD)
5. Standard Deviation
6. Coefficient of Variation (COV)
7. Price-related Differential (PRD)
8. Coefficient of Price-related Bias (PRB)

Range

After ratios are computed and arrayed (put in order from lowest to highest or the opposite); the range can be computed by finding the difference between the highest and lowest ratios. In the first "Level vs Uniformity" example (Example 9), the range was 120% while in Example 10, it was only 32%. Larger ranges generally indicate poorer uniformity, but the frequency of outlying (very low or high) ratios is much more important than mere presence.

The range is the same (190%) in both of the following samples:

Sale #	Ratio Sample A	Ratio Sample B
1	10%	10%
2	10%	100%
3	10%	100%
4	200%	100%
5	200%	100%
6	200%	200%

Uniformity obviously is better in Sample B. Results in sample A are more likely to be indicative of systematic appraisal error.

Frequency Distribution

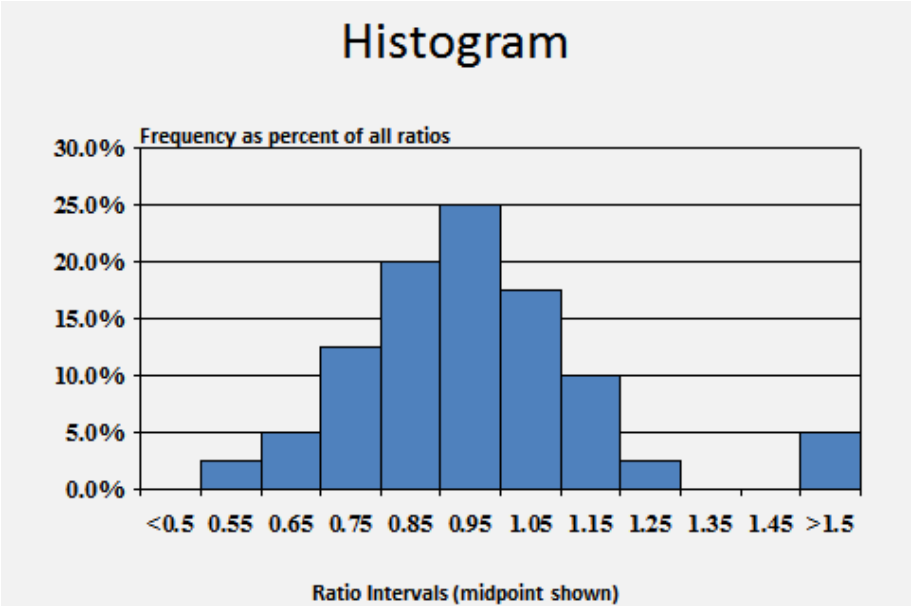
Arrayed ratios can be grouped into brackets typically 5% or 10% in width. A sample frequency distribution follows:

Interval	Number of Ratios	Relative Frequency (Percent of Ratios)
Less than 0.500	0	0.0%
0.500 – 0.599	1	2.5%
0.600 – 0.699	2	5.0%
0.700 – 0.799	5	12.5%
0.800 – 0.899	8	20.0%
0.900 – 0.999	10	25.0%
1.000 – 1.099	7	17.5%
1.100 – 1.199	4	10.0%
1.200 – 1.299	1	2.5%
1.300 – 1.399	0	0.0%
1.400 – 1.499	0	0.0%
More than 1.500	2	5.0%
Total	**40**	**100.0%**

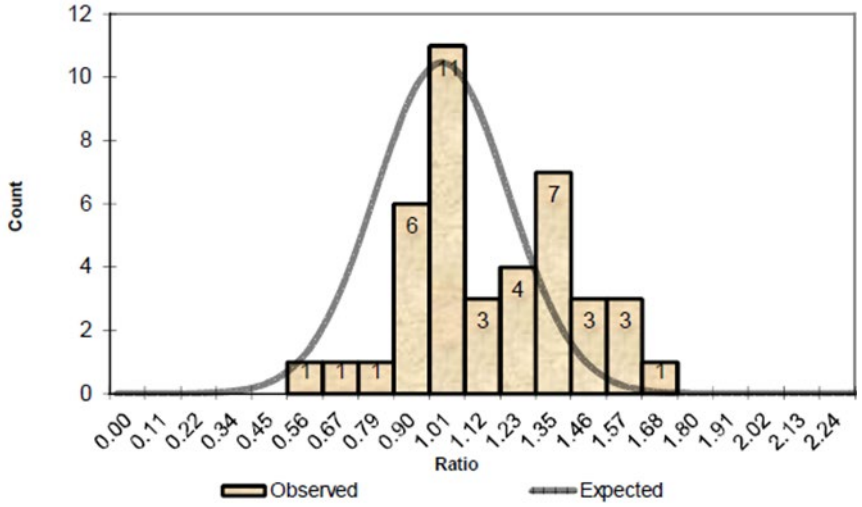
The frequency distribution shows the concentration of ratios within certain brackets or intervals and gives an indication of the degree of uniformity.

Histogram

The histogram represents a picture of the frequency distribution. It is plotted by determining the percent of ratios in each frequency distribution bracket and can be drawn for individual categories or the entire county, as in this plot of the preceding frequency distribution:



Flat histograms or those without central peaks indicate poor uniformity, while the above histogram shows few outliers and good uniformity. By superimposing a histogram on the "normal" curve ("expected" line), the normality of ratio study data can often be ascertained.



Coefficient of Dispersion (COD)

The COD is a direct mathematical measurement of uniformity. It is based on how far each ratio differs from the median and is defined as the average percentage difference between each ratio and the median ratio. The COD is always expressed as a percent of the median and is computed using the following formula:

$$AAD = \frac{\sum \left| A_i/S_i - \widetilde{A/S} \right|}{n} \qquad COD = \frac{100 * AAD}{\widetilde{A/S}}$$

Where: AAD is the average absolute deviation;

$\sum$ means "the sum of";

| | means absolute value disregarding ± sign;

A_i/S_i represents each individual ratio;

$\widetilde{A/S}$ is the median ratio.

COMPUTATION OF THE COD

Example 11:

Assessed Value	Sale Price	Ratio	Difference Between Each Ratio & Median
$ 21,000	$ 25,000	84.00%	16.00%
44,000	50,000	88.00%	12.00%
28,000	30,000	93.33%	6.67%
60,000	60,000	100.00%	0.00%
32,000	30,000	106.67%	6.67%
56,000	50,000	112.00%	12.00%
29,000	25,000	116.00%	16.00%
		Total Difference:	**69.33%**

(AAD) AVERAGE DIFFERENCE = 9.90%

COD = 9.90%

In Example 11, the COD and the AAD were exactly the same. This will occur only if the median is 100%.

COMPUTATION OF THE COD

Example 12:

Assessed Value	Sale Price	Ratio	Difference Between Each Ratio & Median
$ 10,500	$ 25,000	42.00%	8.00%
22,000	50,000	44.00%	6.00%
14,000	30,000	46.67%	3.33%
30,000	60,000	50.00%	0.00%
16,000	30,000	53.33%	3.33%
28,000	50,000	56.00%	6.00%
14,500	25,000	58.00%	8.00%
		Total Difference:	**34.67%**

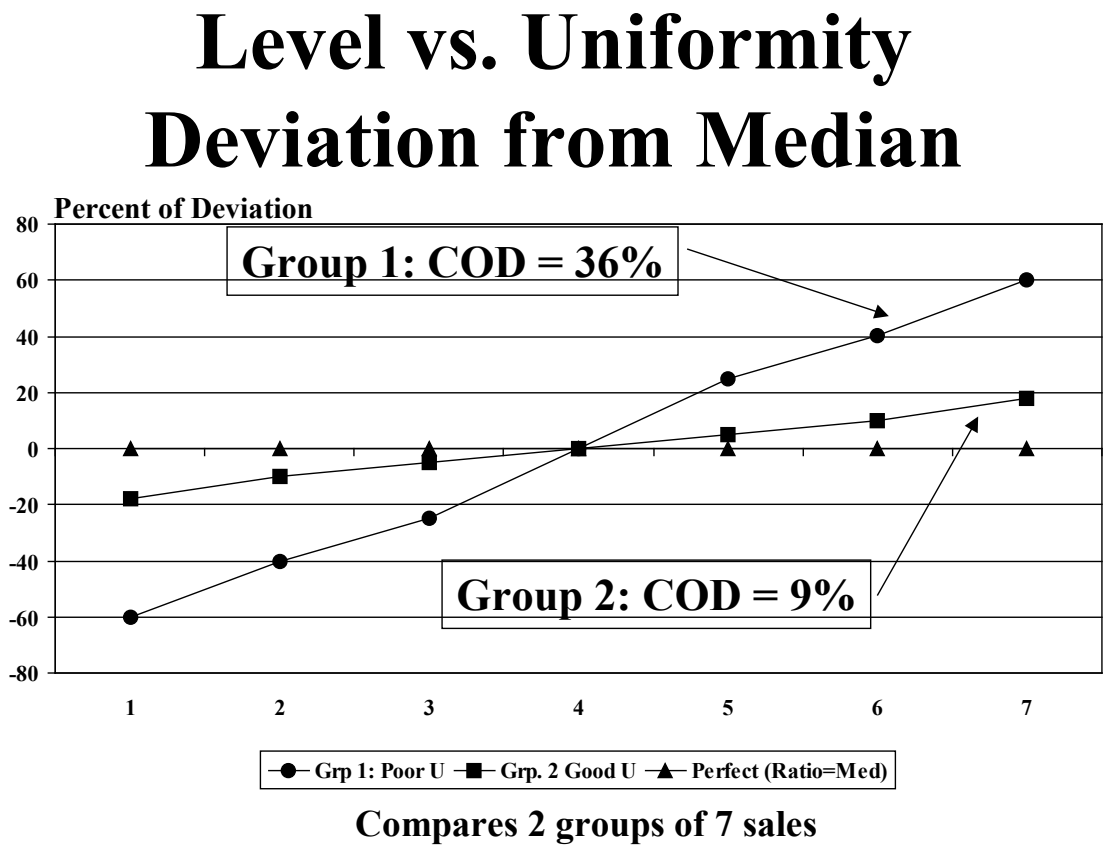
(AAD) AVERAGE DIFFERENCE = 4.95%

COD = 9.90%

Although the median is lower (50%) in Example 12, the relative spread between ratios is the same as in Example 11 and the COD is also the same.

This means that an average difference of 4.95% around a median of 50% represents the same degree of uniformity as an average difference of 9.9% around a median of 100%.

The following chart compares uniformity of two other samples, each consisting of seven sales:



Both groups of sales have exactly the same median and are therefore at the same level. Except for the one sale in each group that is sold for its assessed value (the median was 100% in both groups), all other sales prices deviated from assessments. The difference between the two groups is in the magnitude of the deviation, which was much greater in Group 1, with a COD of 36%, than in Group 2, with a COD of 9%.

To meet standards that have been established for uniformity, the COD must be 15% or less for improved residential property and 20% or less for unimproved property, manufactured housing and commercial property.

A practical grading system for improved property (add 5 points to this scale for unimproved property) would be:

COD (%)	UNIFORMITY CONDITION
< 5	Questionable
5-10	Excellent
10-15	Good
15-20	Somewhat Poor
20-30	Poor
> 30	Very Poor

Since typical marketplace variation precludes perfection in mass (or even single parcel) appraisal, CODs less than 5% are virtually impossible to obtain unless assessments are adjusted to sales prices on individual properties. Three exceptions, in which unusually low CODs may be expected, are:

1. Subdivisions in which lot price is strictly controlled by a developer;
2. Areas in which all improvements are identical or very similar (possibly condominiums);
3. Agricultural land, because of its non-market basis.

The COD is considered the best overall indicator of uniformity, since it is based on the median which is not distorted by high or low ratios. It does not however enable predictions concerning the proportion of property that is assessed within a particular range of the typical assessment level. For example, given a median of 90% and a COD of 10%, we would know that the typical property is assessed between 81% and 99% of market value (a COD of 10% and a median of 90% equates to a ±9 point range around the median ratio). However, we could not estimate the percentage of properties overvalued (100% +) or within any given range (i.e.: 90% - 110%).

Some prediction with the COD may be possible, since, in a normal distribution, the COD may be multiplied by 1.25 to approximate the COV.

Standard Deviation

The portion of property in the population having ratios within a particular range can be predicted from the standard deviation. This statistic is computed based on the unweighted mean ratio. The predictions that are made require the following assumptions:

1. The sample is representative of all property in the category being studied.
2. The sample has been selected randomly.
3. The ratios are normally distributed on either side (high or low) of the mean.

When these assumptions are known to be true the standard deviation, when computed, indicates the following:

Example 13: PREDICTING WITH THE STANDARD DEVIATION.

Given: The mean for all three groups is 95%.

Range of Ratios for indicated portion of property:				
Group	**Standard Deviation (%)**	**68% of prop.**	**95% of prop.**	**99% of prop.**
1	10	85 – 105%	75 – 115%	65 – 125%
2	20	75 – 115	55 – 135	35 – 155
3	30	65 – 125	35 – 155	5 - 185

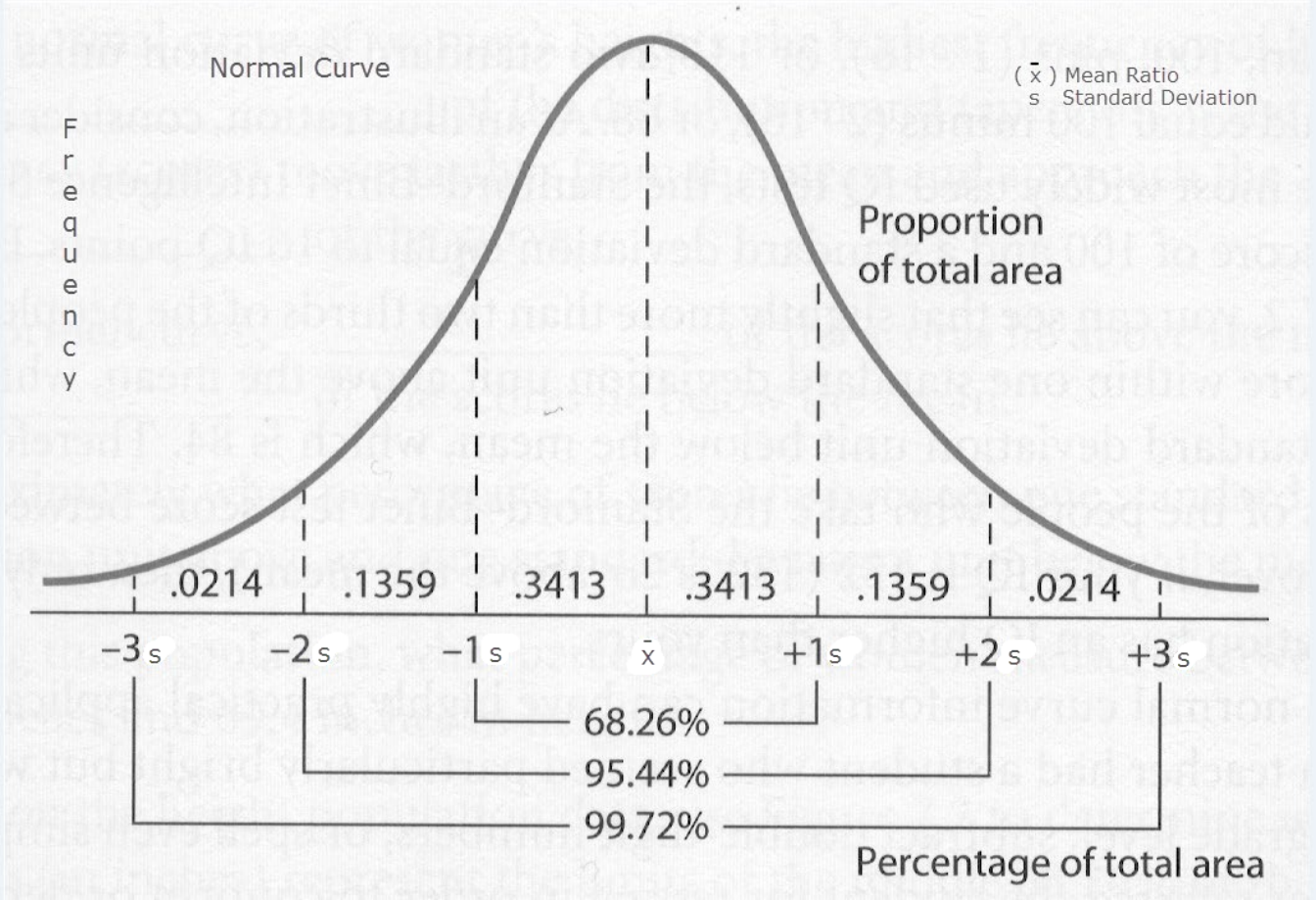
This distribution follows the rule:

# of standard deviation from mean	**% of property found within this range**
± 1	68%
± 2	95%
± 3	99%
± 4	Typically the entire range

In the above example, Group 1 has the best uniformity and can be predicted to have 99% of all property in the category studied assessed between 65% and 125% of market value. In Group 3, with poor uniformity but the same mean assessment level, only 68% of the property is in this range. The other 32% is assumed to be evenly split into higher and lower ratio groups. Therefore, 16% of the property in Group 3 is assessed more than 25% above market value,

while 16% is at least 35% too low (below 65%). In Group 1 only 0.5%(1 property in 200) is assessed above 125% or below 65%. Another illustration of the proportions that can be predicted from the standard deviation is:

Figure 1:



Since the means are 100% in the 2 categories shown in Figure 1, the ranges based on the standard deviations are centered around 100%. If the mean ratio is distorted by outliers, the predictive ability of the standard deviation is diminished. Even if the distribution of ratios is such that the bell-shaped curve illustrated above does not exist, the following predictions are still valid:

NON-NORMAL DISTRIBUTION EXAMPLE	
# of standard deviation from mean	**% of property found in this range**
± 1	Unknown
± 2	75%
± 3	89%

The Coefficient of Variation (COV) is an expression of the standard deviation as a percent of the mean. This "standardizes" the standard deviation so that the COV can be interpreted without additionally knowing the mean. In other words, the COV has exactly the same meaning for a sample with a mean ratio of 40% as for a sample with a mean ratio of 120%.

The standard deviation and COV are computed using the following formulas:

$$s = \sqrt{\frac{\sum (A_i / S_i - (\overline{A/S})^2}{(n-1)}} \qquad COV = \frac{100 * s}{\overline{A/S}}$$

where: s is the standard deviation;

$\sum$ means "the sum of";

n is the number of sales in the sample;

A_i/S_i represents each individual ratio;

$\overline{A/S}$ is the mean ratio.

COMPUTATION OF THE STANDARD DEVIATION & THE COV

Example 14:

Assessed Value ($)	Sale Price ($)	Ratio	Difference Between Each Ratio & Mean	Difference Squared
10,000	25,000	40.00%	-60.00%	3600%
30,000	50,000	60.00%	-40.00%	1600%
22,500	30,000	75.00%	-25.00%	625%
60,000	60,000	100.00%	0.00%	0%
37,500	30,000	125.00%	25.00%	625%
70,000	50,000	140.00%	40.00%	1600%
40,000	25,000	160.00%	60.00%	3600%
Sum of Ratios:		**700.00%**	**Sum of Squares:**	**11650%**

MEAN RATIO: 100.00% Sum of squares divided by sample size -1: 1942%
Square Root: 44.06%

THE STANDARD DEVIATION IS 44.06%

THE COV (COEFFICIENT OF VARIATION) IS
THE STANDARD DEVIATION DIVIDED BY THE MEAN: 44.06%

COMPUTATION OF THE STANDARD DEVIATION & THE COV

Example 15:

Assessed Value ($)	Sale Price ($)	Ratio	Difference Between Each Ratio & Mean	Difference Squared
21,000	25,000	84.00%	-16.00%	256%
44,000	50,000	88.00%	-12.00%	144%
28,000	30,000	93.33%	-6.67%	44%
60,000	60,000	100.00%	0.00%	0%
32,000	30,000	106.67%	6.67%	44%
56,000	50,000	112.00%	12.00%	144%
29,000	25,000	116.00%	16.00%	256%
Sum of Ratios:		**700.00%**	**Sum of Squares:**	**889%**

MEAN RATIO: 100.00% SUM OF SQUARES DIVIDED BY SAMPLE SIZE - 1: 148%
SQUARE ROOT: 12.17%

THE STANDARD DEVIATION IS 12.17%

THE COV (COEFFICIENT OF VARIATION) IS
THE STANDARD DEVIATION DIVIDED BY THE MEAN: 12.17%

In these two examples the following predictions can be made:

Range of Ratios for indicated portion of property:			
Example	**Standard Deviation**	**68% of property**	**95% of property**
14	44.06%	55.94 – 144.06	11.88 – 188.12
15	12.17%	87.83 – 112.17	75.66 – 124.34

Obviously, uniformity is much better in Example 15, where only 5% of all property is predicted to have ratios outside of the range from 75.66% to 124.34% of market value.

The standard deviation is dependent on the mean. A lower mean ratio will result in a lower standard deviation, often giving false indication of better uniformity. To more accurately judge uniformity regardless of the assessment level, the Coefficient of Variation (COV) must be determined as in the following example:

COMPUTATION OF THE STANDARD DEVIATION & THE COV

Example 15a: STANDARD DEVIATION VS. COV

Assessed Value ($)	Sale Price ($)	Ratio	Difference Between Each Ratio & Mean	Difference Squared
10,500	25,000	42.00%	-8.00%	64%
22,000	50,000	44.00%	-6.00%	36%
14,000	30,000	46.67%	-3.33%	11%
30,000	60,000	50.00%	0.00%	0%
16,000	30,000	53.33%	3.33%	11%
28,000	50,000	56.00%	6.00%	36%
14,500	25,000	58.00%	8.00%	64%
Sum of Ratios:		**350.00%**	**Sum of Squares:**	**222%**

MEAN RATIO: 50.00% SUM OF SQUARES DIVIDED BY SAMPLE SIZE - 1: 37%
SQUARE ROOT: 6.09%
STANDARD DEVIATION IS 6.09%

THE COV (COEFFICIENT OF VARIATION) IS
THE STANDARD DEVIATION DIVIDED BY THE MEAN: 12.17%

If Examples 15 and 15a are compared, the standard deviation in 15a is seen to be 1/2 of the standard deviation in Example 15. The COVs however, are identical. The lower standard deviation matches the lower mean (50% vs 100%) in Example 15a. Uniformity relative to the mean is considered identical in these 2 examples.

To meet standards for uniformity the COV (not the standard deviation) must be 20% or less for improved residential property, and 25% or less for unimproved property, commercial property, and manufactured housing.

A practical grading system (add 5 points for unimproved property) would be:

COV (%)	Uniformity Condition
< 5	Questionable
5-10	Excellent
10-20	Good
20-30	Somewhat poor
30-40	Poor
> 40	Very poor

Price-related Differential (PRD)

Property appraisals can sometimes tend to place unequal tax burdens on either high or low value property. Inequity of this type is termed **vertical**, meaning that properties in different value strata are assessed differently in comparison to market value. Assessments would be considered progressive if higher priced property were to be over-assessed in relation to lower priced property. This would occur, for instance, if most $100,000 value properties were appraised at $90,000 (90%) while $30,000 properties were appraised at $24,000 (80%). The opposite situation would be considered regressive.

Manufactured housing is often regressively treated, with older, smaller, lower value properties typically assessed at or over market value, while larger new properties often are under market value.

Bias in favor of high or low priced properties is measured with an index statistic known as the Price-related Differential (PRD). This statistic is computed using the following procedure:

$$\text{PRD} = \frac{\overline{A/S}}{\overline{A}/\overline{S}}$$

where, $\overline{A/S}$ is the mean ratio;

$\overline{A}/\overline{S}$ is the weighted mean ratio.

Three types of results can be demonstrated:			
Price-Related Index	**Meaning**	**Favors**	**Type of Bias**
1.00	Low and High priced property treated same.	Neither	None
> 1.00	Lower ratios on high priced property	High Priced	Regressive
< 1.00	Lower ratios on low priced property	Low Priced	Progressive
Standard: 0.98 – 1.03 = OK			

If the PRD is between 0.98 and 1.03, the degree of bias or vertical tax inequity is not considered significant. However, the Mann-Whitney and Price-Related Bias (PRB) tests described following this section are considered more definitive and should be used in addition to the PRD.

The PRD is computed by dividing the mean by the weighted mean. This calculation effectively measures the distortion in the weighted mean caused by high or low ratios on high or low valued property. Since the mean is not affected by value, but only by ratios, this measurement can serve as a baseline for the comparison. The following examples show the computation of the PRD and demonstrate the tax inequities represented:

PRD COMPUTATION CHART
EXAMPLES OF VARIOUS CONDITIONS:

Example A:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 25,000	$ 20,000	125.00%
2	24,000	30,000	80.00%
3	31,000	40,000	77.50%
4	40,000	50,000	80.00%
5	60,000	60,000	100.00%
6	79,000	70,000	112.86%
Totals:	**259,000**	**270,000**	**575.36%**

WEIGHTED MEAN = 95.93%
MEAN = 95.89%
PRD = 1.00*
*DOES NOT FAVOR LOW OR HIGH PRICED

Example B:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 30,000	$ 20,000	150.00%
2	40,000	30,000	133.33%
3	45,000	40,000	112.50%
4	50,000	50,000	100.00%
5	40,000	60,000	66.67%
6	45,000	70,000	64.29%
Totals:	**250,000**	**270,000**	**626.79%**

WEIGHTED MEAN = 92.59%
MEAN = 104.46%
PRD = 1.13**
**FAVORS HIGH PRICED

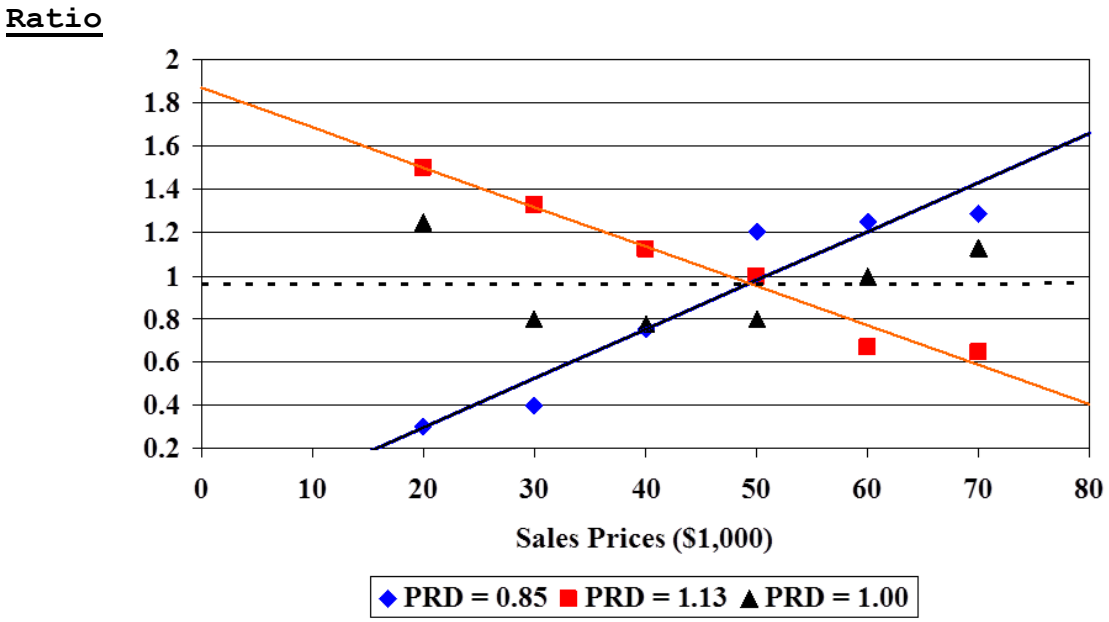
PRD COMPUTATION CHART
EXAMPLES OF VARIOUS CONDITIONS:
Example C:

Sale #	Assessed Value	Sale Price	Ratio
1	$ 6,000	$ 20,000	30.00%
2	12,000	30,000	40.00%
3	30,000	40,000	75.50%
4	60,000	50,000	120.00%
5	75,000	60,000	125.00%
6	90,000	70,000	128.57%
Totals:	**273,000**	**270,000**	**518.57%**

WEIGHTED MEAN = 101.11%
MEAN = 86.43%
PRD = 0.85***
***FAVORS LOW PRICED

These examples use the same group of sales and show the effects of different assessments. Although no group is assessed perfectly, there is no discernable distortion based on value in group A. In group B, however, assessment ratios clearly decline as value (sale price) increases; the assessments favor higher price property. The opposite occurs in group C.

Graphically, the 3 examples appear as follows:



Alternate Statistical Tests for Vertical Equity

Mann-Whitney Test

As a measure of the degree of value related equity problems, the PRD has certain drawbacks. In small samples, the PRD is very sensitive to distortion caused by the presence of a very small number of "outlier" type sales. One or two high value, low ratio sales (or vice versa) can easily result in a PRD which appears to indicate a significant value related problems. However, the significance of these results may not be provable in a statistical sense. When this is the case, value related inequity may not be occurring even though the PRD does not meet standard. Similarly, in very large samples, the PRD may be within acceptable standards, yet there may still be value related appraisal problems occurring in a small sector of the properties. Perhaps, for example, appraisals are low on homes over $500,000, but this group constitutes only 15 sales in a sample of 800. If vertical equity has been achieved in all other strata, it is unlikely that the PRD will indicate any problem.

To guard against inaccurate judgments and decisions based on the PRD, ratio studies can be developed to test specific values strata. In addition, the significance of value related inequity can be tested using statistical tests, such as the Mann-Whitney test, which can be used to compare the rank of the ratio corresponding to each sale to each sale price. If there is no value related inequity, these ranks will be relatively randomly dispersed. If most of these high-priced sales have low or high ranks, value related inequity is indicated.

The Mann-Whitney test is calculated by finding *U* from the following formula and then testing the statistical significance with a *z* score.

$$U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \mathrm{R}_1$$

Where: n_1 is the number of sales in the group < the mean sale price,
n_2 is the number of sales in the group > the mean sale price
R_1 is the sum of the ranks in the group < the mean price.

Once *U* is calculated, the *z* score is determined using the following formula:

$$z = \frac{U - (n_1 n_2)/2}{\sqrt{(n_1 n_2)(n_1 + \mathrm{n}_2 + 1)/12}}$$

If *z* is greater than 1.96 or less than –1.96, there is a statistically significant difference between the ratios in the two groups and value related inequity is likely. The Mann-Whitney test is demonstrated in Appendix III g. For this test to be used, the

smaller group must have no fewer than 8 ratios. (See PAAA, IAAO, 1990 and Mass Appraisal of Real Property, IAAO 1999 for additional restrictions.)

Coefficient of Price-related Bias (PRB)

Regardless of the significance of the PRD or the Mann-Whitney test, the results can only be interpreted qualitatively and cannot be interpreted to provide the degree of vertical inequity, only its presence and direction (ie: favors high or low priced). The PRB is a statistic that enables interpretation of the magnitude of vertical inequity. This statistic has been incorporated into the IAAO 2013 *Standard on Ratio Studies.*

The PRB is obtained by regressing percentage difference from the median ratio on percentage differences in value. The PRB indicates how far assessment ratios rise or fall when values (ie: a term that reflects ½ sale price plus ½ assessed value) double. For example, a PRB of 0.05 means that assessment ratios increase 5% every time values double.

The PRB has another advantage over the PRD – it is relatively insensitive to value outliers.

By way of a standard for this statistic, a 95% confidence interval around the coefficient should include some part of the range from -.05 - + .05. If such an interval fails to include some part of the range from -.10 - + .10, vertical inequity should be considered unacceptable.

By way of qualitative comparison, it is notable that categories seldom fail the PRB with sufficient confidence to warrant a conclusion of vertical inequity. The opposite is true of the PRD, but conclusions using that statistic are based strictly on the point estimate as there is no corresponding confidence interval. This is a distinct weakness in the PRD, which may lead to false "positive" conclusions of vertical inequity.

STATISTICAL MEASURES OF RELIABILITY

All of the statistics previously discussed represent measurements made on sample data. In fact, by definition, statistics always concern samples. The ultimate purpose of all of the measurements, however, is to determine assessment conditions for the entire group or population of properties in each category studied.

Measurements made directly on populations provide parameters or facts. Since we must indirectly measure the population from a limited sample, based only on properties that sell, we are forced to estimate the population parameters. The precision and validity of this estimation is based on several factors including:

1. Sample Randomness: Sample acquisition should be unbiased with every property having an equal chance for selection. Although there is no direct bias in the way we choose sales to be used, each property probably does not have an equal opportunity to sell and get into the ratio study. This requirement should be considered partially met by our sampling procedure.

2. Sample Representativeness: This requires that the sample be drawn from the population under investigation and that individual observations (types of properties) occur in the sample in approximately the same frequency as in the population. Stratification by area and category of property helps fulfill this requirement. However, there may be a tendency in some unsegregated areas for extensive sales activity in one subdivision and few or no sales in another. If the property characteristics and other market influences are similar in the two areas, there is no problem. However, if the areas' economic forces differ significantly, the requirement for representativeness may not be fully satisfied.

3. Normality: This is the requirement for assessment ratios to be randomly distributed with respect to the mean throughout both the sample and population. Many sources consider this doubtful in regard to assessment ratios. Usually, however, only large ratio study samples can be proven not normal with any significant degree of certainty. Non-parametric statistics may be employed to avoid inaccurate parameter estimates that may otherwise result in non-normal situations.

Measuring Ratio Study Reliability

The reliability of ratio study results is the most important single aspect of the ratio study. Decisions made using ratio studies have the potential of affecting hundreds of thousands of taxpayers across the state. If the results used as a basis for these decisions are not reliable, two types of errors can occur:

1. Category values may be adjusted (up or down) when, in fact, no adjustment is warranted.
2. Category values may be considered satisfactory when, in fact, adjustments should be made.

These errors are equally serious in terms of taxpayer inequity and may be made by either the state though equalization decisions or the county through appraisal decisions.

Given a reasonably random and representative sample, reliability depends on two factors:

1. Sample size
2. Sample uniformity

Large, uniform samples produce more reliable results than small samples with poor uniformity.

Ratio study reliability can be measured using two different approaches:

1. Direct measurement of probability of true mean between 90% and 110% (or any selected level);
2. Indirect measurement of range within which true mean or median is likely to be found.

The second of these approaches involves the development of 95% (or other appropriately significant) confidence intervals based on the:

1. Mean,
2. Median, or
3. Weighted mean.

Regardless of the base statistic (mean, median, weighted mean), the confidence interval determined in a ratio study indicates the range within which we are 95% (or any other selected degree of certainty) certain that the true assessment level occurs.

In other words, a confidence interval of 85% - 115% indicates that we are 95% confident that if all property in the category being studied were to be sold and assessment ratios computed, the true overall average level of assessment would be between 85% and 115%.

The following chart illustrates the effects of sample size and uniformity (COV) on the mean-based confidence interval (the mean is assumed to be 100% in each sample shown):

Sample Size	Coefficient of Variation (COV) (%)	95% Confidence Interval (CI)
5	10	87.6 – 112.4
10	10	92.8 – 107.2
50	10	97.2 – 102.8
100	10	98.0 – 102.0
5	20	75.2 – 124.8
10	20	85.7 – 114.3
50	20	94.5 – 105.5
100	20	96.1 – 103.9
5	30	62.8 – 137.2
10	30	78.5 – 121.5
50	30	91.7 – 108.3
100	30	94.1 – 105.9

Note: Confidence intervals developed for determining compliance with assessment level standards will be slightly narrower, because 90% (or, in some cases, 80%) intervals, rather than 95% intervals, are being determined. Also, these intervals are based on the median and therefore will only coincidentally be symmetrical.

Computation of Probability

Ratio study probabilities are computed to determine the chance that sale of all properties in a category within a jurisdiction would indicate a particular mean level of assessment. For ratio study standards, a desirable range for the true level of assessment is between 90% and 110% of market value. We therefore calculate the probability that this level has been attained. This probability is calculated using:

1. the "t" test for samples of 30 or fewer sales;
2. the "z" test for larger samples.

The formula for z or t is as follows:

$$\text{z or t} = \frac{\overline{\text{A/S}} - \mu}{\text{s} / \sqrt{n}}$$

where: $\overline{A/S}$ is the sample mean ratio;
μ ("mu") is the population mean to be tested;
s is the sample standard deviation;
n is the sample size.

Once this equation is solved, standard tables (Appendices IV and V) must be consulted to determine the probability corresponding to the computed t or z value. The following examples demonstrate this procedure:

Example 16:
A sample of 36 sales has a mean ratio of 85% and a standard deviation of 10%. We wish to determine with a 5% maximum error the probability that the true mean level of assessment is between 90% and 110% of market value.

Since this probability question requires determining the probability that the true mean is within a given range, two separate calculations must be done:

1. Find the probability that μ >90% by: $z = \frac{.85 - .90}{.10/\sqrt{36}} = -3.0$

2. Find the probability that μ >110% by: $z = \frac{.85 - 1.10}{.10/\sqrt{36}} = -15.0$

Looking up -3.0 in the z table in Appendix IV indicates that there is a 0.13% probability that the true mean (μ) exceeds 90%. Looking up -15 indicates that there is virtually no probability that the true mean exceeds 110%. Therefore, the probability that the true mean is between 90% and 110% must be 0.13%. If we decide that assessment level is unacceptably low and values should be increased, there will be a 0.13% probability that the mean level was already acceptable.

Example 17:
A sample of 81 sales has a mean ratio of 87% and a standard deviation of 18%. We wish to determine the probability of a true mean between 90% and 110%.

1. the z score for μ > 90%: $z = \frac{.87 - .90}{.18/\sqrt{81}} = -1.50$

2. the z score for μ > 110%: $z = \frac{.87 - 1.10}{.18/\sqrt{81}} = -11.50$

From the z table, the probability that the true mean is greater than 90% is 6.68%. There is virtually no probability that the true mean is greater than 110%. Therefore, the probability that the true mean is between 90% and 110% is 6.68%.
Probabilities cannot exceed 100% or absolute certainty. However, the standard in use in Idaho requires 5% or higher probability that the

true mean is in the 90% to 110% range, for samples using the mean to determine compliance. This means that, for such categories, county determined assessed values will be considered market value provided that there is at least a 5% chance that an overall 90% to 110% range has been attained and provided the sample mean has not been outside of the 90% to 110% range previously (see Standards).

With reference to examples 16 and 17, assessment level would be considered satisfactory in Example 17, but unsatisfactory in Example 16 (See: Standards and Equalization section).

Computing Confidence Intervals

The Mean Confidence Interval

The following formula can be used to compute this confidence interval:

$$CI(\overline{A/S}) = \overline{A/S} \pm \left[\frac{(t)*(s)}{\sqrt{n}}\right]$$

Where: CI $\left(\overline{A/S}\right)$ is the confidence interval around the mean;

t is the constant from the appropriate column of the "t table" (Appendix V) based on n-1 degrees of freedom; (Note: the column to be used depends on the selected probability of the mean being outside of the interval - to be 95% sure the mean is within the interval, select the .05 probability column.)

s is the sample standard deviation;

n is the sample size.

In calculating 95% confidence intervals using the means and other information in examples 16 and 17, we find:

Example 16:

Mean = 85%, Standard deviation = 10%, sample size = 36.

$$95\%\ \text{CI} = .85 \pm \left[\frac{(1.96)*(.10)}{\sqrt{36}}\right]$$

$$= .85 \pm .03$$

$$= (.82 - .88)$$

We can be 95% confident that the true mean level of assessment is between 82% and 88% of market value.

Example 17:
Mean = 87%, standard deviation = 18%, sample size = 81.

$$95\%\,\mathrm{CI} = .87 \pm \left[\frac{(1.96)*(.18)}{\sqrt{81}}\right]$$

$$= .87 \pm .04$$

$$= (.83 - .91)$$

We can be 95% confident that the true mean level of assessment is between 83% and 91% of market value. For compliance testing, 1-tailed 95% intervals must be computed. These would use constants from the 0.1 probability column in Appendix V. For Example 17 this would mean substituting 1.645 for the 1.96 constant. This would change the confidence interval to .87 ±.03 or .84 - .90, which would barely meet standard. Except when noted for compliance testing purposes, confidence intervals shown in this manual are based on two-tailed computation methods. These intervals indicate a range within which, with 95% certainty, the true mean will lie.

In general, narrower confidence intervals indicate greater reliability and occur when large samples with good uniformity are available. The effects of sample size and uniformity on unweighted mean based confidence intervals are shown in the following examples:

CONFIDENCE INTERVALS Ratio Study Samples for Categories A through D				
Statistics	**A**	**B**	**C**	**D**
Mean	95	95	95	95
Standard Deviation	10	50	50	16
Sample Size	100	100	16	4
95% Confidence Interval (mean based)				
*UCL	96.96	104.8	121.6	120.5
*LCL	93.04	85.2	68.4	69.5
Width	3.92	19.6	53.2	51.0

In these 4 examples "UCL" indicates the upper confidence limit while "LCL" shows the lower limit of the interval. The width is the difference between these upper and lower limits. Sample A is the most reliable while sample C is the least reliable.

The Weighted Mean Based Confidence Interval

This interval provides information similar to that given by the unweighted mean based interval. Only the central point and calculation process have been changed. This interval is important when measuring reliability in any sample having a PRD significantly greater or less than 1.00.

The 95% confidence interval using the weighted mean can be found from the following formula:

$$95\%\ \mathrm{CI}\left(\overline{A}/\overline{S}\right) = \overline{A}/\overline{S} \pm t_{\propto .05} * (S[\overline{A}/\overline{S}]),$$

where: $\overline{A}/\overline{S}$ is the weighted mean,

$t_{\propto .05}$ is a constant from the .05 error (probability) column of the t-table using n-1 degrees of freedom,

$S[\overline{A}/\overline{S}]$ is the standard error of the weighted mean.

The formula for is: $S[\overline{A}/\overline{S}]$

$$S\left[\overline{A}/\overline{S}\right] = \frac{\sqrt{\sum \overline{A^2} - 2\ (\overline{A}/\overline{S}\sum(A*S) + (\overline{A}/\overline{S})^2(\sum S^2)}}{\overline{S}\sqrt{(n)\ (n-1)}}$$

In the above formula, terms are used as follows:

A is assessed value;
S is sale price;
$\overline{S}$ is average sale price.

The following example demonstrates the use of this formula:

Example 18: Weighted Mean Confidence Intervals
Note: all values are expressed in thousands of dollars.

Sale	Assessed Value (A)	(A^2)	Sale Price (S)	(S^2)	(A) * (S)
1	10	100	40	1,600	400
2	20	400	30	900	600
3	30	900	30	900	900
4	30	900	25	625	750
5	25	625	19	361	475
6	20	400	12	144	240
Totals:	**135**	**3,325**	**156**	**4,530**	**3,365**

The mean ratio is 101.7% while the weighted mean is 86.54%. The PRD is 1.17. Terms to be substituted into the formula are the following:

n = 6

$\overline{S}$ = 156/6 = 26 (average sales price)

$2(\overline{A}/\overline{S})\sum(A*S) = 2(0.8654)(3365) = 5,824.142$

$(\overline{A}/\overline{S})^2(\sum S^2) = (0.8654)^2(4530) = 3,392.595$

With 5 degrees of freedom the t constant = 2.571 (Appendix V) Substituting and solving the equation gives the following:

$$95\%\text{CI}(A/S) = 0.8654 + 2.571 * \frac{\sqrt{3325 - 5824.142 + 3392.595}}{26 * \sqrt{6 * 5}}$$

$$= 0.8654 \pm 2.571 * \left\{\frac{29.89}{142.41}\right\}$$

$$= 0.8654 \pm 0.5396 = 0.3258 - 1.4050$$

The mean-based interval for this example is 49.00% - 154.40%, substantially different from the weighted mean based interval above.

The Median Confidence Interval:

Reliability of results for samples or populations which are not normally distributed can best be tested by developing a median based confidence interval. This statistic is determined by a process whereby certain ratios are selected and represent lower or upper bounds on the interval. Outlying low or high ratios do not affect this selection process, which is based strictly on the number of ratios (sales) available in the sample. The procedure is taken from the IAAO, PAAA textbook and follows:

Median Confidence Interval Formula

Depending on even or odd count, one of two formulas is used to calculate the rank of the ratios corresponding to the upper and lower confidence interval limits.

If "n" (sample size) is even:

$$j = \left[\frac{1.96 * \sqrt{n}}{2} + 0.5 \right]$$

If "n" is odd:

$$j = \frac{1.96 * \sqrt{n}}{2}$$

where, j is the number of ratios to be counted up and down from the median to determine the rank of the upper and lower confidence interval. Before counting, the result (j) must always be rounded upward to the next integer.

Odd Example 19:

If n = 25, which is odd,

$$j = \frac{1.96 * \sqrt{25}}{2} = 4.9$$, which is rounded to 5

Since the median is the 13th ratio, the lower limit of the median confidence interval would be the 8th ratio (13-5) and the upper limit would be the 18th ratio (13+5).

Even Example 20:

If n = 16, which is even,

$$J = \frac{1.96 * \sqrt{16}}{2} + 0.5 = 4.42,$$ which is rounded to 5

Since the median is between the 8th and 9th ratios, we count **down** 5 from the ratio above the median and **up** 5 from the ratio below the median. This gives ranks of 9 - 5, or 4 for the lower limit, and 8 + 5, or 13 for the upper limit.

Because of errors which may occur if the lowest and highest ratios are determined to be one or both of the confidence interval limits, this test is only considered valid for sample sizes of 9 or more. However, binomially based methods have been developed and published and allow 90% median confidence intervals to be computed more accurately for smaller samples. These procedures were incorporated into state conducted ratio studies beginning in 2007.

DETERMINING NORMALITY

Reliability testing methods based on the unweighted mean ratio depend on normal distribution of the sample. How closely a given sample distribution fits a normal distribution can be determined using various procedures as listed below:

Method	Sample Size For Use
Chi - Square (χ^2)	100 or greater
Binomial Approximation	25 - 99
Direct Binomial Test	Less than 25

The Chi Square Procedure

For this procedure, it is necessary to create a frequency distribution for the sample using at least 6 brackets or intervals. An expected frequency for the number of ratios that should fall within each of the chosen brackets can then be developed, assuming a perfectly normal distribution. Brackets should be chosen so that the expected frequency is at least 5 ratios in each bracket. For every 5 brackets, it is permissible for 1 to have an expected frequency of fewer than 5. If the expected frequency were much smaller (near 0), the presence of even a single ratio actually falling in the bracket

would support a conclusion of a non-normal distribution. Therefore lower and upper end brackets should be combined and widened to prevent this distortion.

After expected frequencies have been computed and brackets finalized, observed (actual) frequencies and expected (hypothetical) frequencies are compared and the probability that the sample distribution is normal can be determined. Example 21 demonstrates this statistic:

Example 21:

sample size n = 114

sample average ratio $\overline{A/S}$ = 100.58

sample standard deviation s = 15.22

Bracket K

Equal to or			Expected Frequency (E)	Observed Frequency (O)
Greater than	but	Less than		
0%		80%	10.04	9
80%		90%	17.50	9
90%		100%	27.51	33
100%		110%	28.18	38
110%		120%	19.04	15
120%		130%	8.47	6
130%			3.05	4

Observed frequencies were taken directly from the ratio in the sample of 114 sales. Expected frequencies were calculated (see z test calculations following the χ^2 calculation). Chi-square is calculated as follows:

$$X^2 = \frac{\Sigma(O-E)^2}{E} =$$

$$\frac{(9-10.04)^2}{10.04} + \frac{(9-17.50)^2}{17.50} + \frac{(33-27.51)^2}{27.51}$$

$$+ \frac{(38-28.18)^2}{28.18} + \frac{(15-19.04)^2}{19.04} + \frac{(6-8.47)^2}{8.47} + \frac{(4-3.05)^2}{3.05} = 10.63$$

(χ^2 here is pronounced "chi-squared" and Σ means "the sum of" and is pronounced "sigma".)

Expected frequencies are determined by using the z test and finding the probable number of ratios in a given bracket assuming normality. For example, for the 80% - 90% bracket, the following calculation is used:

Probability of A/S (any ratio) less than 80%:

$$z = \frac{A/S - \overline{A/S}}{s} = \frac{80 - 100.58}{15.22} = -1.35$$

The probability that any ratio is less than 80% is determined from the z table and here is 8.85%.

$$z = \frac{A/S - \overline{A/S}}{s} = \frac{90 - 100.58}{15.22} = -0.70$$

The probability that A/S is less than 90% is 24.20%.

The probability of a given ratio between 80% and 90% is therefore 24.20% - 8.85%, which equals 15.35%. This percent (15.35%) multiplied by 114 ratios (n) results in an expected 17.5 ratios in the specified bracket, assuming a normal distribution.

Normality is always the assumed state, unless we have sufficient evidence to prove that a distribution is non-normal. From the chi-square table (Appendix VI), we can find that to reject our hypothesis of a normal distribution with 95% confidence, the calculation would have to result in a chi-square statistic of 12.59 (There are 7 brackets or 6 degrees of freedom). Since our answer was 10.63 and this does not exceed the critical value of 12.59, we must conclude that the distribution of ratios in our sample is normal.

In addition to allowing us to determine normality, the chi-square calculation process shows clearly the ratio brackets within which the actual results (observed frequencies) differ the most from the expected, "normal" results. In the sample used in Example 21 the greatest differences occurred in the 80% - 90% bracket. The deviation in this bracket accounts for nearly one-half (4.13) of the total chi-square value (10.63) as follows:

$$X^2 = \frac{(O - E)^2}{E}$$ (No $\sum$, since this is for one bracket)

For the 80% to 90% bracket, E = 17.50 and O = 9, therefore,

$$X^2 = \frac{(17.50 - 9)^2}{17.50} = 4.13$$

Non-normal determinations are often caused by an absence of ratios that are very low or very high. It is a common misconception that ratios which deviate significantly from the average occur only in non-market value sales. This error may result in exclusion of many sales which might enhance reliability and permit a conclusion of normality. The following chart demonstrates typical ratio ranges that would be anticipated in normally distributed samples.

Typical Large Sample (n >100) Ratio Range, Mean ($\overline{A/S}$) = 100

Uniformity (Comments)	Standard Deviation	Expected Range (%)
Excellent	8	68 - 132
Very Good	12	52 - 148
Good	15	40 - 160
Borderline	20	20 - 180

Note: the above chart reflects the principle that the range is ordinarily approximated by the mean ±4 standard deviations. For small samples, this is reduced to ±3 standard deviations:

Typical Small Sample Range (Mean = 100%)	
Standard Deviation (%)	**Expected Range**
8	76 - 124
12	64 - 136
15	55 - 145
20	40 - 160
30	10 - 190

Binomial testing procedures are more appropriate for samples with less than 100 ratios; results are similar to those in Example 21.

Binomial Test of Normality

$$z = \frac{0.5(n-1) - x_s}{\sqrt{0.25n}}$$

Samples composed of between 25 and 99 ratios may be tested for normality by using a binomial approximation method employing the following formula:

Where: n is the total number of ratios;

x_s is the number of ratios in the smaller of the two groups:

1. Number of ratios greater than the mean, or
2. Number of ratios less than the mean.

z is the score to be tested against a critical value to determine normality.

Example 22:

Sample size (n) = 25

Mean $\left(\overline{A/S}\right)$ = 100%

Standard Deviation (s) = 22.8%

Array of Ratios:

Sale #	Ratio (%)	Sale #	Ratio (%)
1	80	14	93
2	82	15	93
3	83	16	95
4	85	17	97
5	85	18	98
6	86	19	99
7	86	20	101
8	86	21	109
9	87	22	137
10	89	23	145
11	90	24	150
12	92	25	159
13	93		

Number of ratios exceeding the mean: 6
Number of ratios less than the mean: 19

Using the data from the preceding example, where x_s would be 6 (the number of ratios in the smaller group) and n equals 25, we would calculate:

$$Z = \frac{0.5(25-1)-6}{\sqrt{.25(25)}} = 2.4$$

The critical value of z necessary to reject the hypothesized normality at the 95% confidence level is 1.96 (a constant used for this purpose).

Since our answer was greater than 1.96, we must conclude that the sample is not normally distributed. Since the standard deviation was 22.5% in Example 22, we would have expected a range of ± 67.5 points (3 standard deviations) corresponding to ratios from 32% to 168%. Since we are much closer to meeting the upper limit of this range, it appears very likely that sales with even moderately low ratios may have been deleted from the sample.

In non-normal distributions, the median based statistics are usually the best indicators of assessment conditions. The median is 93% in Example 22 and probably gives a truer picture of level than the mean in this case.

Direct determination of normality

For samples with less than 25 ratios, probability statements concerning normality can be derived directly from binomial tables. The following table shows the maximum number of ratios that may occur in the smaller of the group of ratios, greater or less than the mean, and still permit us to conclude that we have a normal distribution.

Binomial Table	
Sample Size (n)	**Critical Value: Normal if number of ratios in small group is at least:**
5 or less	N/A
6	1
7	1
8	2
9	2
10	2
11	3
12	3
13	4
14	4
15	4
16	5
17	5
18	6
19	6
20	6
21	7
22	7
23	8
24	8
25	8

The critical values in this table correspond to probabilities in actual binomial tables. See for example, Table 3 on page 614 of the IAAO Textbook, Property Appraisal and Assessment Administration (PAAA).

By converting probabilities to critical values, we are able to reject the normal distribution hypothesis only when we can be at least 95% sure that a given distribution is not normal. For example, if a sample of 14 sales had four ratios above the mean and ten below, we

would not be sufficiently confident to reject the normal distribution hypothesis. We would conclude that the sample did not deviate sufficiently from a normal distribution to support a non-normal conclusion. However, the actual probability of normality is only 9% in this case. In other words, in this and other tests of normality we are assuming normality. We will not conclude otherwise unless the evidence for the alternative conclusion is overwhelming and the chance of incorrectly concluding that we do not have a normal distribution is less than 5%.

Sample normality does not necessarily indicate population normality. In fact, much assessment literature specifically ascribes non-normal distributions to assessment ratio populations. However, even in non-normal populations, the central limit theorem holds that the distribution of means is nearly normal. This supposition requires only that there be a random and normally distributed probability of occurrence of a different mean if a second sample were selected. The second mean should be normally distributed in reference to the initial sample mean.

EDUCATION AND ASSISTANCE

The State Tax Commission plans and conducts an ongoing series of regional instructional workshops designed to help assessment personnel understand and utilize the statistical methods presented in this manual.

THE RATIO STUDY AS AN APPRAISAL TOOL

Although ratio studies commonly are perceived as measuring level, uniformity (horizontal equity), and regressivity (vertical equity), they also can help with:

- ** identification of general assessment needs;
- ** identification of specific assessment needs;
- ** development of trend factors and time adjustments;
- ** measurement of appraiser performance;
- ** identification of assessment biases
  such as: class, location, construction type, age, value;
- ** defense of property values and appraisal technique in
  appeals.

In addition, the statistics involved may be employed to help with the development of depth tables, economic rent and expenses, land schedules, depreciation tables, local cost modifiers, correlation of value estimates, etc.

By performing ratio studies against values derived using each of the cost, income, and sales comparison approaches, plus possibly a combination figure, you can study the results and know the best appraisal approach and value estimate.

Another example of using statistics in appraisal would be in analyzing what is economic rent. First, array the data and figure the mean, median, and mode of the group. Remember we don't want an average rent that might not reflect any property, we want a typical rent. Somewhere near the mean, median, and mode is the answer. By using uniformity statistics such as the COV and COD against several of the best estimates, a statistically supported number for economic rent can be obtained.

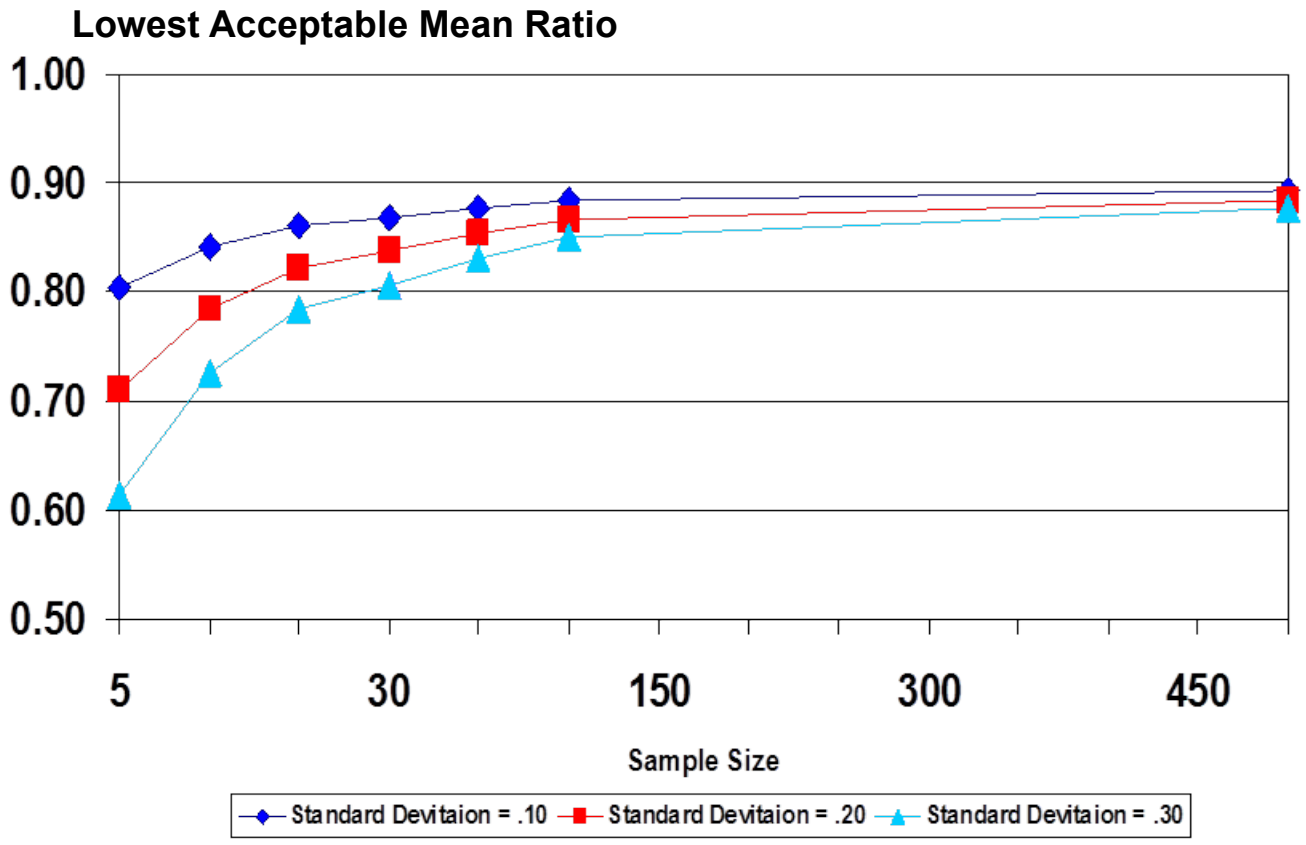
The big key to meaningful results is the proper stratification of data into groups with a lot of similarity. The results from studying these groups are very valuable and consequently can increase the equity of the property tax.

1. Assessment Level

The overall assessment level within any category of property will be considered to be market value provided the applicable ratio study measure of central tendency is within ±10% of 100%. This test will be considered met in the 2023 ratio studies if the **median** based 90% (two tailed) confidence interval includes some part of the range between 90% and 110%, provided that the category did not fail this test using 80% confidence intervals during both of the most recent two ratio studies for the category.

Final compliance status will be determined using 80% confidence intervals for any categories that passed the most recent two ratio studies only by using 90% confidence intervals.

The following Chart shows examples of lowest acceptable sample means given various sample sizes and standard deviations:



Note: The preceding chart is provided to illustrate compliance ranges, which actually will be based on medians and therefore cannot be mathematically predicted.

2. Assessment Uniformity

a. Coefficient of Dispersion (COD):

Uniformity is considered adequate if this measure does not exceed 15% in improved residential property categories. Dispersion of up to 20% is allowed in vacant property categories, manufactured housing, and commercial property categories.

b. Coefficient of Variation (COV):

Assessment uniformity is considered adequate if this measure does not exceed 20% in improved residential property categories. In vacant property categories, manufactured housing, and commercial property categories, variation of up to 25% is acceptable.

c. Price-related Differential (PRD):

Results in the 0.98 to 1.03 range are considered satisfactory.

Note: Measurements of uniformity of less than 5% are considered questionable, probably indicating non-representative sample results. Results in this range should not be considered meaningful indicators of assessment uniformity.

STATE TAX COMMISSION EQUALIZATION PROCEDURE

The 2023 ratio study done by the State Tax Commission for equalization purposes is to be completed in March 2024. Prior to completion, consulting appraisers are to review each study to ensure that the data has been properly categorized and proper adjustments made. Upon completion, each county assessor is notified of the compliance status and complete statistical analysis of each category of property tested.

Each category found to be out of compliance with assessment level standards is subject to additional testing with a follow-up study typically using calendar year 2023 sale prices time-adjusted to January 1, 2024. These sale prices are compared against 2024 assessments as soon as these are available.

The results of the follow-up study are subject to corroboration using sales occurring through June, 2024. If the follow-up study is considered valid and representative, and indicates that adjustments made by the assessor have resulted in compliance with assessment level standards, the category is considered in compliance and, barring subsequent action by the county board of equalization that results in non-compliance, no ratio study based county equalization recommendations would be made to the State Tax Commission in 2024.

If a valid and representative follow-up study indicates non-compliance with assessment level standards, equalization recommendations will be developed on the basis of this follow-up study. If the follow-up study cannot be done or is otherwise considered invalid or non-representative, equalization recommendations may be made on the basis of the original ratio study or a corroborative study.

The State Tax Commission may delay implementation of any equalization adjustment for one year, if there is reason to question the representativeness of the ratio study.

Regardless of the need for county equalization adjustments, railroad values may be subject to equalization if commercial and industrial property ratio studies conducted within the area of any railroad show level of assessment for that area that is provably below 95%, based on the weighted mean.

Prior to the meeting of the State Tax Commissioners as a Board of Equalization beginning on the second Monday of August each year, and following notification of county assessors and county boards of equalization, the State Tax Commission staff will prepare equalization recommendations for each county for categories of property still considered out of compliance with assessment level standards, in accordance with the following general procedure:

STAFF POSITION	FUNCTION
Property Tax Policy Bureau Chief	Presents statistical information from Ratio Study. Compares results to Ratio Study Standards and makes recommendations for equalization.
Consulting Appraiser	Presents judgment and opinion concerning the assessment conditions and the sample used for the Ratio Study. May concur with statistically based recommendation or may present substitute recommendation based on additional information or findings regarding validity or representativeness of the ratio study or any follow-up study.
Division Administrator	Reviews all information and meets with Consulting Appraiser and Property Tax Policy Bureau Chief to decide on final recommendation to be submitted to State Board of Equalization.

Staff recommendations are not finalized until information has been compiled for all counties and categories subject to ratio study analysis.

The preceding process occurs during the period from March through July. Other than the Commissioner who supervises the Ad Valorem section, no state tax commissioner is involved in this staff recommendation procedure.

The Division Administrator finalizes staff recommendations for the State Board of Equalization and dates are established for the tax commissioners to hear these recommendations during the State Board of Equalization meeting. Counties with equalization recommendations are then notified of the specific recommendations to be presented. County officials are invited to be present at a formal meeting. At this meeting the Division Administrator presents the staff recommendations and the Property Tax Policy Bureau Chief presents supportive information regarding these recommendations. County officials then have an opportunity to present any additional pertinent information for consideration.

The state tax commissioners take all recommendations under advisement and issue equalization decisions by the fourth Monday of August. Decisions are reached by voting commissioners, with the Commissioner who supervises the Property Tax Division chairing the Board and participating in a non-voting, advisory capacity. Deliberations and final decisions are made in open meetings that may be attended by county officials and the public.

Provided there is sufficient time to give affected property owners legally required notice and appeal rights, at any time during this process, until State Tax Commission (state board of equalization) decisions are finalized, county commissioners may request permission from the State Tax Commission to reconvene the County Board of Equalization to change assessed values to comply with ratio study standards.

Case I: Unacceptable Level (category by county)

This situation can occur if the 90% two-tailed confidence interval around the sample median ratio fails to overlap any of the 90% - 110% range or if the 80% two-tailed confidence interval around the sample median ratio fails to overlap any of the 90% - 110% range, provided that the most recent two ratio studies for the category also failed at this degree of confidence.

If unacceptable level is determined for a category for which the ratio study sample is considered non-representative or invalid, the county will be notified of non-complying status, but no trending recommendation will be prepared or submitted to the State Board of Equalization. If the level of assessment can be ascertained despite an inadequate follow-up ratio study, adjustment recommendations may be prepared and submitted to the State Board of Equalization.

The following procedures use the timeline for the 2024 equalization program (2023 ratio studies and 2024 Board of Equalization).

If unacceptable level is determined for a category for which the sample is considered representative (or a category that was **not** in compliance when last studied, but for which the sample had been considered non-representative), a trending recommendation will be prepared and submitted to the State Board of Equalization, unless:

a. the county can prove that adjustments equal to those about to be ordered have already been made, or

b. market value changes, demonstrated by as many new sales as are available through June, 2024, prove that the adjustments are no longer necessary, or

c. a validated follow-up ratio study shows that 2022 assessments were in compliance.

Each follow-up ratio study must be validated using procedures outlined in steps 1 through 4 below. The procedure in step 1 may also be used to demonstrate market changes, which may indicate compliance despite non-complying results in the original and follow-up ratio studies. Ratio study validation steps follow:

1.) Sales from October, 2023 through June, 2024 compared to 2024 assessments. These sales must be time-adjusted to January 1, 2024.

2.) A statement indicating trends or procedures used to correct assessed values, including the number of parcels or identification of areas adjusted.

3.) A list of assessed values before and after application of adjustments. This list must be created for all, or a randomly selected sample of, non-selling properties within the areas adjusted within the non-complying category. The list must show the percent change between 2023 and 2024 assessed values for each property selected and must show a total percent change for the non-selling properties.

4.) A comparison of 2023 and 2024 total assessed value for the non-complying category.

If adequate proof is provided that county adjustments to non-complying categories have resulted in a median level of assessment that indicates compliance with assessment level standards, state adjustments will not be recommended. If, however, county adjustments alter assessment level, but do not produce results which comply with level standards, modified recommendations will be developed and submitted to the State Board of Equalization. If state ordered trending is necessary, the following procedure is used for staff recommendations:

1. Trending is to be by category (except as outlined in item 3 below) based on the median assessment ratio calculated for the category.

2. The trending factor is computed by dividing the chosen measure of level **into** 100.

3. Provided there is sufficient time to give affected property owners legally required notice and appeal rights, at any time during this process prior to State Board of Equalization action, a county may request that its board of equalization be reconvened for the purpose of trending by area or sub-category rather than by category of property. All new values must be submitted to the State Tax Commission for review.

4. Any category trended by the state in a given year will be considered in compliance with assessment level standards for that year, once the trend is applied.

Case II: Unacceptable Uniformity.

The county is notified of any category with unacceptable uniformity, but equalization action is not indicated in these cases.

Case III: Unacceptable Level within Railroad areas.

Adjustments to railroad values are computed when commercial and industrial ratio studies in counties to which a particular railroad's values are to be apportioned, fail to have an upper limit of the 90% confidence interval around the weighted mean that is at least 95%. If this failure occurs, the railroad's Idaho value will be adjusted downwards by the difference between the weighted mean ratio (point estimate) and 100%. This reduction will be done prior to apportionment. This same process may be applied to other operating property as well.

RATIO STUDY REPRESENTATIVENESS

As has been discussed elsewhere, there are three distinct purposes for state ratio studies:

1. They are used to provide information to assist county assessors in their annual assessment programs.

2. They are used to evaluate uniformity to determine any potential need for the State Tax Commission to complete the county appraisal program.

3. They are used to assist the State Tax Commission in its task of equalizing assessments by category of property statewide.

The State Tax Commission is involved in the equalization process for several reasons including the following:

1. To ensure compliance with a federal requirement under the 4R Act for centrally assessed railroad property to be at (within ±5%) the same level as certain commercial and industrial properties.

2. To ensure that taxpayers in joint taxing districts are assessed at the same level regardless of their county of residence.

3. To ensure that taxpayers in different categories of property within one district are assessed equitably.

4. To maintain general equity between all centrally assessed property and locally assessed property.

To accomplish these tasks, it is critical that the state board of equalization be given the most reliable and credible information available regarding assessment level.

Review of assessed values and changes in assessed values of non-selling properties is one way of monitoring the representativeness of ratio study information.

DEFINITIONS

All terms are defined in accordance with their usage in this report.

Arithmetic Mean (average) ($\overline{A/S}$):

The result of dividing the sum of ratios in a series by n, the number of ratios in a sample. A measure of central value. Simple to compute, but may be disproportionately influenced by extreme ratios. Also known as: unweighted mean, mean.

$$\text{Mean} = \frac{\sum A/S}{n} = \overline{A/S}$$

Where A/S = each individual ratio and n = the number of ratios in a sample.

Array:

An ordered series of ratios from low to high or high to low.

Binomial Test:

A procedure for determining whether ratios follow a normal distribution. Used if fewer than 100 sales are available in a sample.

Category:

Category means those types of property defined by numbers given in rule 130 and indicated on the county abstract of valuation.

A category is a specific type of property represented numerically on the county abstract of valuation.

The term also refers to the combinations typically used in the ratio study. Example: 20/41, urban residential lots with improvements. Depending on use, the term may refer to primary or secondary categories.

Chi-Square Test:

A test by which the closeness of fit of sample data to a normal distribution may be determined.

$$\chi^2 = \frac{\Sigma(O-E)^2}{E}$$ where

χ^2 is the chi-square statistic,
O is the observed frequency, and
E is the expected frequency.

Confidence Interval:

An interval or range computed from a sample. This interval enables estimation of a population mean or median with a known degree of error. The bounds of the confidence interval are known as the upper confidence limit (UCL) and the lower confidence limit (LCL).

Dispersion, Coefficient of (COD):

The primary measure of assessment uniformity. It is based on the median and expresses uniformity in terms of the average difference between each ratio and the median. The COD is shown as a percent of the median.

Frequency Distribution:

An arrangement of ratios that groups this data to show how often given ratio ranges occur. (see: Histogram)

Geometric Mean:

A measure of assessment level determined by multiplying all of the values in a series together and then taking the "n"th root of this product.

$$\text{geometric mean} = \left(A_1/S_1 * A_2/S_2 * A_3/S_3 * \ldots A_n/S_n \right)^{1/n}$$

where A_n/S_n represents each ratio in the sample;
and n = the number of ratios.

Histogram:

A pictorial representation of a frequency distribution. (see: Frequency Distribution)

Hypothesis:

In inferential statistics this is a statement about which the truth or validity is to be tested. The usual procedure is to state what one chooses to accept in the absence of sufficient evidence to the contrary (the statement is called the "null hypothesis"), specify the relationship or statement to be proved (termed the "alternative hypothesis"), and analyze the available data to determine whether the null hypothesis can be rejected (and hence the alternative hypothesis accepted) at some confidence level.

Level:

A general expression of the overall relationship between assessed values and sales prices. Measured by determining:

1. Arithmetic mean
2. Median
3. Geometric mean
4. Weighted mean

Mean: (see: Arithmetic Mean or Weighted Mean or Geometric Mean)

Median $\left(\widetilde{A/S}\right)$:

The middle ratio in an array. The rank of the ratio to be selected can be found by:

median rank = .5(n) + .5, where n = the number of ratios.

Also, equivalent to the 50th percentile.
(see: Dispersion, Coefficient of)

Mode:

The mode is the most frequently occurring number in a sample. If two or more numbers occur with equal frequency and no other number is present in greater frequency, there will be more than one mode.

Normal Distribution:

A type of frequency distribution which is symmetrical and approximately bell-shaped.

Population:

The group of all items or properties in a particular category from which a sample is drawn.

Price-related Bias (PRB):

A statistic that indicates by how much ratios tend to fall or rise as values double. This statistic allows interpretation of the magnitude of vertical inequities.

Price-related Differential (PRD):

This statistic measures the treatment of property in relation to high or low value. Tendencies to value high priced property disproportionately with regard to low priced property can be identified.

The PRD is calculated by finding a weighted mean ratio by dividing the sum of the assessed value for all sales by the sum of the sale prices for all sales. This quotient is then divided into the unweighted mean ratio, with the result being the PRD.

Ratio (A/S):

The result of dividing the assessed value of any property by its sale price. The answer is usually multiplied by 100 to be expressed as a percent.

Reliability:

Any measurement of the likelihood that sample results equal population results. Specifically, an attempt to determine whether the average assessment level measured using a ratio study sample corresponds to the true overall average assessment level on all properties in any category.

Weighted Mean:

An average ratio derived from the total assessed value and total sale price in an entire sample. This measure is subject to distortion if price-related assessment bias exists. (See: PRD).

Sample:

Sample means the sales which will be subject to ratio study analysis to reach a conclusion or make a recommendation relative

to an abstract category of property in a county or in a specific area within a county.

Standard Deviation (s):

A statistical measure of the spread or distance of ratios from the mean in a sample.

$$s = \sqrt{\frac{\sum (A_i / S_i - (\overline{A/S})^2}{(n-1)}}$$

Where:

$\sum$ means "the sum of";

n is the number of sales in the sample;

A_i/S_i represents each individual ratio;

$\overline{A/S}$ is the mean ratio.

Standard Error of the Mean:

A statistic that indicates the probable magnitude of difference between a result (the arithmetic mean) obtained from a sample and the actual result if measured for a population as a whole.

$$SE_{\overline{A/S}} = \frac{\left(\frac{s}{\sqrt{n}}\right)}{\overline{A/S}}$$

where $SE_{A/S}$ = standard error of the mean $(\overline{A/S})$,

s = standard deviation,
and n = number of ratios in the sample.

(see: Standard Deviation)

Uniformity:

An expression of the equity of property taxes to the taxpayers within any given category of property. Determined by the following measurements:

1. Coefficient of Dispersion (COD)
2. Coefficient of Variation (COV)
3. Price-related Differential (PRD)

Variation, Coefficient of:

A measure of the spread of sample ratios from the mean expressed as a percent of the mean. An expression of the standard deviation in terms relative to the mean.

$$COV = \left(\frac{s}{\overline{A/S}} \right) * 100$$

where s = standard deviation

$\overline{A/S}$ = the arithmetic mean.

(see: Standard Deviation)

APPENDICES

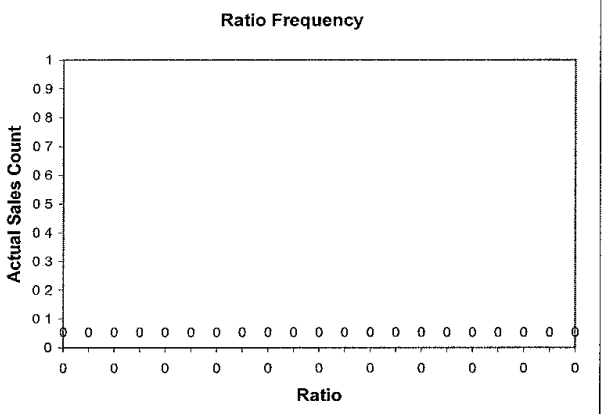
APPENDIX I a – Blank Template Sample

#N/A		#N/A	Observed Sales Dates:	
FINAL Study when Initialed and dated.	Using 2006 Assessed Values	Assessment Date: 01/01/2006	From: 01/00/1900	To: 01/00/1900
			Selected Time Period:	
			10/01/2005	09/30/2006

SAMPLE STATISTICS	
Sample size (n)	0
Total Assessed Value	#DIV/0!
Total Sales Price	#DIV/0!
Mean Assessed Value	#DIV/0!
Mean Sales Price	#DIV/0!
Standard Deviation AV	#DIV/0!
Standard Deviation SP	#DIV/0!
Median Assessed Value	#NUM!
Median Sales Price	#NUM!
ASSESSMENT LEVEL	
Arithmetic Mean Ratio	#DIV/0!
Median Ratio	#NUM!
Weighted Mean Ratio	#DIV/0!
Geometric Mean Ratio	#NUM!
UNIFORMITY	
<Extreme> Lowest Ratio	0.00%
Highest Ratio	0.00%
Coefficient of Dispersion	#DIV/0!
Standard Deviation	#DIV/0!
Coefficient of Variation	#DIV/0!
Price-related Differential	#DIV/0!

RELIABILITY		
90% Confidence Intervals:	Lower	Upper
Around the Mean	#DIV/0!	#DIV/0!
BINOM - Around the Median	#N/A	#N/A
Around the Weighted Mean	#DIV/0!	#DIV/0!
Probability True Mean 90-110	#DIV/0!	
80% Confidence Intervals:	Lower	Upper
Around the Mean	#DIV/0!	#DIV/0!
BINOM - Around the Median	#N/A	#N/A
Around the Weighted Mean	#DIV/0!	#DIV/0!

NORMALITY	
Chi Square Test	
Computed Value	N/A
Critical Value	N/A
Conclusion:	
Binomial Test	
# ratios below mean:	#N/A
# ratios above mean:	#N/A
z:	#N/A
Conclusion:	#N/A
#N/A	
Mann-Whitney Test	#DIV/0!
#DIV/0!	
Kurtosis	#DIV/0!
Skew	#DIV/0!



Compliance Checks:

Level:

MEAN	90% Confidence Interval:	#DIV/0!
MEAN	80% Confidence Interval:	#DIV/0!

Uniformity:

COD:	#DIV/0!
COV:	#DIV/0!
PRD:	#DIV/0!

COMMENTS:

Secondary Category(ies) with sales

Count	Category	Description

Remarks:

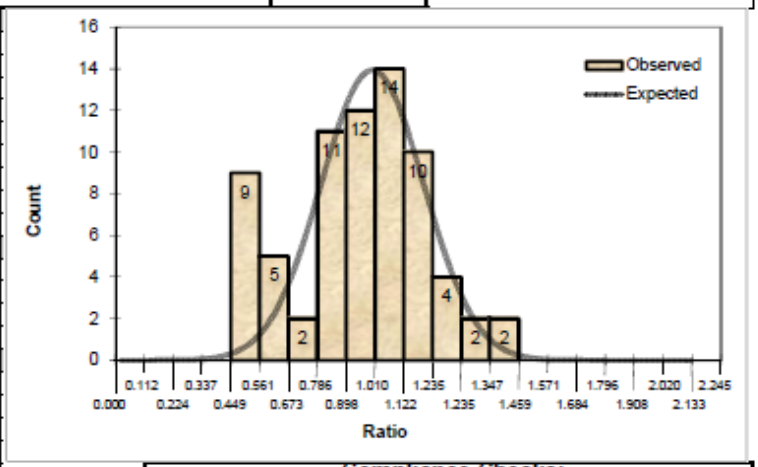
Reviewer's Initials

Appendix I b – Completed template example

County	Primary Group: Vacant Residential (Ref ID 1)	Combined Categories	TIME PERIOD STUDIED	
RATIO STUDY Not Official until 'FINAL' dated & initialed	Using 2015 Assessed Values	Assessment Date: 01/01/2015	From: 10/01/2014	To: 09/30/2015
Sales Price is Time Adjusted	Time Adjustment Considered, but not Applied.			

SAMPLE STATISTICS	
Sample size (n)	71
Total Assessed Value	$2,814,260
Total Adjusted Sales Price	$3,064,725
Mean Assessed Value	$39,637
Mean Adjusted Sales Price	$43,165
Standard Deviation AV	$27,571
Standard Deviation SP	$26,609
Median Assessed Value	$37,951
Median Sales Price	$37,000
ASSESSMENT LEVEL	
Arithmetic Mean Ratio	95.23%
Median Ratio	97.44%
Weighted Mean Ratio	91.83%
Geometric Mean Ratio	91.27%
UNIFORMITY	
Lowest Ratio	46.50%
Highest Ratio	151.58%
Coefficient of Dispersion	21.57%
Standard Deviation	26.25%
Coefficient of Variation	27.57%
Price-Related Bias	-0.0099
Price-Related Differential	1.04

T-Score: -0.2841



RELIABILITY	Lower	Upper
90% Confidence Intervals:		
Around the Mean	90.10%	100.35%
Around the Median	91.84%	103.06%
Around the Weighted Mean	85.60%	98.06%
Around the COD	17.68%	26.62%
Around the PRB	-0.0780	0.0583
Probability True Mean 90-110	95.25%	
80% Confidence Intervals:	Lower	Upper
Around the Mean	91.23%	99.22%
Around the Median	92.69%	102.09%
Around the Weighted Mean	86.97%	96.68%
NORMALITY Test Results:	Normal	
Chi Square Test	N/A	
Binomial Test	Normal	
Mann-Whitney Test	-1.6974	
Significance of Value Related Inequity - Weak		
D'Agostino-Pearson	Normal	
Shapiro-Wilk W	N/A	
Kurtosis (2 - 5) = 2.43	Acceptable	
Skew (-0.5 - 1) = -0.14	Acceptable	

COD Standard	Maximum
Primary Group: Vacant Residential (Ref ID 1)	20.00%

Compliance Checks:

Level:	Compliance Met?	
90% Confidence Interval:	☐ YES	☐ NO
80% Confidence Interval:	☐ YES	☐ NO

Uniformity:	COD Standards met?	YES
	COD:	Poor
	COV:	Poor
	PRD:	Favors High Priced
	PRB:	Meets IAAO Standard, No Significant Bias

COMMENTS:

2.74% of the originally available population has been trimmed.
Outliers Reviewed using Std. Score 2 Std Dev.
Below 0.3849 (1 sales) and Above 1.526 (1 sale)

Secondary Category(ies) with sales

Count	Category	Description
2	12	Unimproved Rural Res Tract
30	15	Unimproved Rural Res Sub
2	18	Unimproved Other Land
37	20	Unimproved Urban Res Lot/Acre

COV/COD	
Expected	125%
Observed	128%

PRELIMINARY

Date

Analyst

Time Adjustment Data Sheet

Inflationary Market Pattern

COUNTY
2041 Properties

NOTE: The 'SLOPE' function is affected by extreme data points.
Please review the data carefully.

Original Rate	Residual Rate	Time Adjustment Statistics	
-1.45%	0.02%	<== Quarterly Averages	0.00018
-1.80%	-0.34%	<== Quarterly Medians	-0.00340
-1.50%	0.00%	<== Slope Results	-0.00002 High Confidence

Note: If results indicates > 1% per month Time Adjustment -- PLEASE REVIEW

Selected Category Observed Sale Dates: 10/1/2005 to 9/1/2006

TARGET DATE: 1/1/2006 (Zero Months)

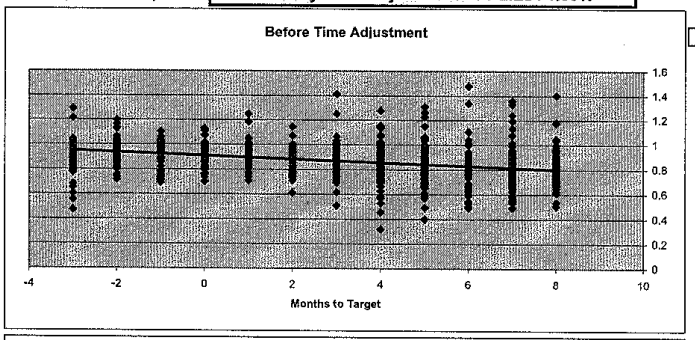
Monthly Time Adjustment APPLIED: 1.50%

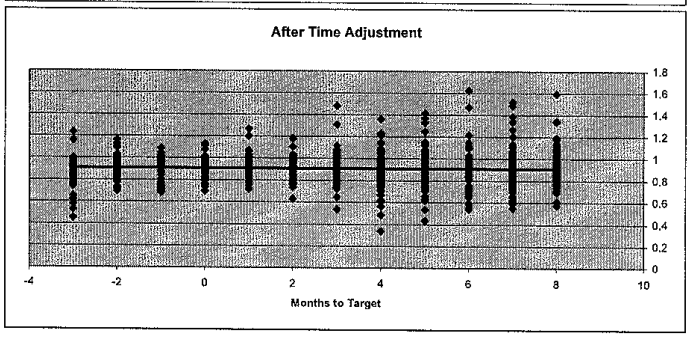
Sales Ratios

	Counts	Average	Median
1st Qtr.	265	0.9051	0.9274
2nd Qtr.	201	0.9252	0.9431
3rd Qtr.	328	0.9022	0.8997
4th Qtr.	320	0.9065	0.8990

Sale Prices (Unadjusted)

	Counts	Average	Median
1st Qtr.	265	$ 133,753	$ 115,000
2nd Qtr.	201	$ 126,314	$ 112,000
3rd Qtr.	328	$ 123,077	$ 109,900
4th Qtr.	320	$ 133,675	$ 120,000





2041 Properties

Only Categories with sales are listed

Count	Category	Description
1114	2041	Improved Urban Res

Appendix II

APPENDIX III a

SALES DATA RECORD

Study 1: Improved Residential (20/41)

($)

	Assessor's Market Value				
Sale #	Land	Imp.	Total	Sales Price ($)	Ratio(%)
1	5,995	23,980	29,975	54,500	55.00
2	11,036	44,144	55,180	89,000	62.00
3	16,570	48,030	64,600	95,000	68.00
4	5,140	14,740	19,880	28,000	71.00
5	14,001	55,047	69,048	95,900	72.00
6	4,736	18,944	23,680	32,000	74.00
7	6,080	24,320	30,400	40,000	76.00
8	8,453	33,812	42,265	53,500	79.00
9	3,910	15,640	19,550	23,000	85.00
10	4,150	15,582	19,732	20,771	95.00
11	5,300	21,200	26,500	25,000	106.00
12	15,000	13,200	28,200	20,000	141.00

Study 2: Residential Land (20)

Sale #	Market Value	Sales Price ($)	Ratio(%)
1	6,400	8,000	80.00
2	10,080	12,000	84.00
3	9,660	10,500	92.00
4	3,906	4,200	93.00
5	5,820	6,000	97.00
6	7,425	7,500	99.00
7	11,330	11,000	103.00
8	9,360	9,000	104.00

APPENDIX III b1

Worksheet #2
Frequency Distribution, Relative Frequency and Histogram

Instruction: (Refer to your Sales Data Record)

1. List number of ratios falling within each interval separately for each category.
2. Total frequencies for both categories.
3. Using the combined total column only, divide the number of ratios in each interval by the total number of ratios.
4. Convert each answer to a percent by multiplying by 100. This is the relative frequency.
5. Transfer all information from this worksheet to the histogram sheet.
6. Draw the histogram.

Frequency Distribution:

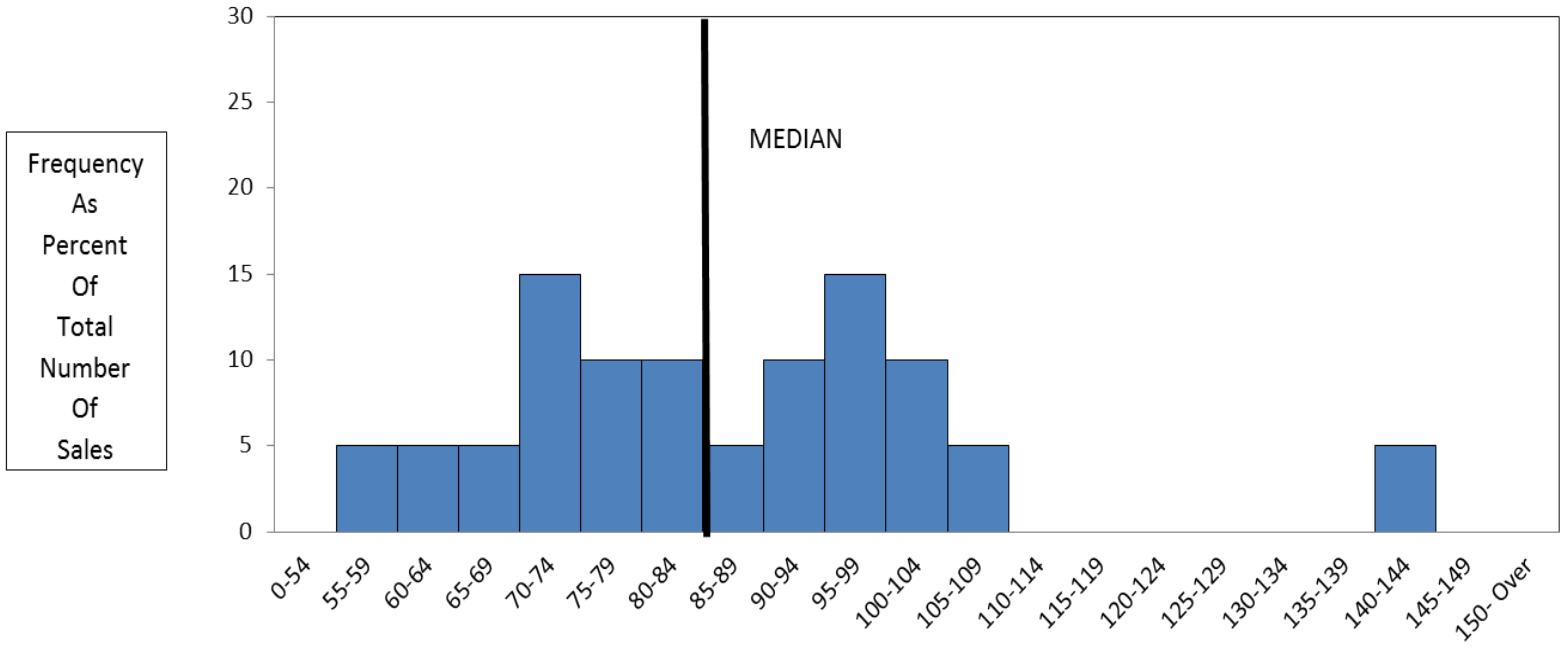
% Ratio Interval	Frequency (Fi)		Frequency (%) (Fi/N) x 100	
	Imp. Resid. (20/41)	**Resid. Land (20)**	**Combined Total**	
Less than 55%	-	-	-	
55 - 59	1	-	1	5
60 - 64	1	-	1	5
65 - 69	1	-	1	5
70 - 74	3	-	3	15
75 - 79	2	-	2	10
80 - 84	-	2	2	10
85 - 89	1	-	1	5
90 - 94	-	2	2	10
95 - 99	1	2	3	15
100 - 104	-	2	2	10
105 - 109	1	-	1	5
140 - 144	1	-	1	5
Total	**XXXXXX**	**XXXXXX**	**20**	**100**

Appendix III b2

RATIO STUDY – SAMPLE FREQUENCY DISTRIBUTION AND HISTOGRAM

20__ Ratio Study 20__ Sales 20__ Assessments Summary for _____ County

Category	No. of Sales	Frequency of Ratios of Assessed Value to Sales Price/Appraisal Value																				
		0 54	55 59	60 64	65 69	70 74	75 79	80 84	85 89	90 94	95 99	100 104	105 109	110 114	115 119	120 124	125 129	130 134	135 139	140 144	145 149	150 Over
Resid. w/imp	12	-	1	1	1	3	2	-	1	-	1	-	1	-	-	-	-	-	-	1	-	-
Resid. Land only	8	-	-	-	-	-	-	2	-	2	2	2	-	-	-	-	-	-	-	-	-	-
Totals	20	-	1	1	1	3	2	2	1	2	3	2	1	-	-	-	-	-	-	1	-	-



APPENDIX III c1

Coefficient of Dispersion

Computation Worksheet #3

Purposes: Using the data from your Sales Data Record, complete the following charts and compute the coefficient of dispersion (COD) for each category of property.

Improved Residential Category (20/41)			
A	**B**	**C**	**D**
Sale #	**Ratio**	**Median**	**Difference between Median and Ratio**
1	55.00	75.00	20.00
2	62.00	75.00	13.00
3	68.00	75.00	7.00
4	71.00	75.00	4.00
5	72.00	75.00	3.00
6	74.00	75.00	1.00
7	76.00	75.00	1.00
8	79.00	75.00	7.00
9	85.00	75.00	10.00
10	95.00	75.00	20.00
11	106.00	75.00	31.00
12	141.00	75.00	66.00
Total:			**180.00**

APPENDIX III c2

Worksheet #3 (Continued)

Residential Land Category (20)			
A	**B**	**C**	**D**
Sale #	**Ratio**	**Median**	**Difference between Median and Ratio**
1	80.00	95.00	15.00
2	84.00	95.00	11.00
3	92.00	95.00	3.00
4	93.00	95.00	2.00
5	97.00	95.00	2.00
6	99.00	95.00	4.00
7	103.00	95.00	8.00
8	104.00	95.00	9.00
Total:			**54.00**

APPENDIX III c3

Worksheet #3 (Continued)

Steps for Above Charts:

1. In Column B, list ratios from Sales Data Record.

2. Determine the median ratio for each category and show this figure in Column C next to each ratio shown in Column B.

3. Subtract the median from Column B ratio. Ignore the sign (+) of the answer (this gives us the absolute value of the difference) shown in Column D.

4. Add up Column D for each chart and show the total. Treat all numbers as if they were positive.

5. Determine the average absolute deviation by dividing the total difference from Column D by n (the number of sales). (Use space provided below to show calculation for each category.)

$$\frac{\text{Total Difference}}{n} = \text{Average absolute deviation}$$

Imp. Resid. Category:

$$\frac{180.00}{12} = 15.00$$

Resid. Land Category:

$$\frac{54.00}{8} = 6.75$$

APPENDIX III c4

Worksheet #3 (Continued)

6. Divide your answers from step 5 by each median and multiply by 100 to find the Coefficient of Dispersion (COD) expressed as a percent. (Use space provided below.)

$$\frac{\text{Average Absolute Deviation}}{\text{median}} \times 100 = \text{COD}$$

Imp. Resid. Category:

$$\frac{15.00}{75.00} \times 100 = 20.00$$

Resid. Land Category:

$$\frac{6.75}{95.00} \times 100 = 7.11$$

Conclusions:

Compare uniformity in the two categories.

The improved residential category demonstrates somewhat poor uniformity, while there is excellent uniformity shown by the residential land sample.

APPENDIX III d1

Worksheet #4

Standard Deviation and Coefficient of Variation

Purpose: Using the data from your Sales Data Record, complete the following charts and compute the standard deviation and coefficient of variation for each category of property.

Improved Residential Category (20/41)				
A	**B**	**C**	**D**	**E**
	Ratio	**Mean Ratio**	**Difference**	**(Difference)^2**
Sale #	(A/S)*100	$\overline{A/S}$	$\left(A/S - \overline{A/S}\right)$	$\left(A/S - \overline{A/S}\right)^2$
1	55.00	82.00	-27.00	729.00
2	62.00	82.00	-20.00	400.00
3	68.00	82.00	-14.00	196.00
4	71.00	82.00	-11.00	121.00
5	72.00	82.00	-10.00	100.00
6	74.00	82.00	-8.00	64.00
7	76.00	82.00	-6.00	36.00
8	79.00	82.00	-3.00	9.00
9	85.00	82.00	3.00	9.00
10	95.00	82.00	13.00	169.00
11	106.00	82.00	24.00	576.00
12	141.00	82.00	59.00	3,481.00
Totals:	**984.00**			**5,890.00**

APPENDIX III d2

Worksheet #4 (Continued)

Improved Residential Category (20/41)				
A	**B**	**C**	**D**	**E**
	Ratio	**Mean Ratio**	**Difference**	**(Difference)^2**
Sale #	(A/S)*100	$\overline{A/S}$	$\left(A/S - \overline{A/S}\right)$	$\left(A/S - \overline{A/S}\right)^2$
1	80.00	94.00	-14.00	196.00
2	84.00	94.00	-10.00	100.00
3	92.00	94.00	-2.00	4.00
4	93.00	94.00	-1.00	1.00
5	97.00	94.00	3.00	9.00
6	99.00	94.00	5.00	25.00
7	103.00	94.00	9.00	81.00
8	104.00	94.00	10.00	100.00
Totals:	**752.00**			**516.00**

APPENDIX III d3

Worksheet #4 (Continued)

Procedure:

1. In Column B of each chart, list ratios from sales data record.

2. Total Column B ratios.

3. Divide each Column B total by n to get the mean ratio ($\overline{A/S}$). Show results below.

   Imp. Resid. mean $\frac{984}{12} = 82.00\%$

   Resid. Land mean $\frac{752}{8} = 94.00\%$

4. Show the mean ratio in Column C next to each Column B ratio.

5. Subtract the mean ratio from each Column B ratio and show the result in Column D.

6. Square each Column D result and show in Column E.

7. Total the numbers in Column E.

8. Divide the total of Column E by n-1. Show result below:

   Imp. Resid.: $\frac{\text{Col. E. Total}}{n-1} = \frac{5890}{11} = 535.45$

   Resid. Land: $\frac{\text{Col. E. Total}}{n-1} = \frac{516}{7} = 73.71$

   (This is known as the variance.)

9. Take the square root of the figure obtained in Step 8. This is the standard deviation. Show result below:

   Imp. Resid.: $\sqrt{535.45} = 23.14$

   Resid. Land: $\sqrt{73.71} = 8.59$

APPENDIX III d4

Worksheet #4 (Continued)

10. Divide the standard deviation by the mean (Step 3) and multiply your result by 100 to get the coefficient of variation.

$$\frac{s}{A/S} \; x \quad 100 = COV$$

Imp. Resid.: $\frac{23.14}{82.00}$ x 100 = (COV) 28.22%

Resid. Land: $\frac{8.59}{94.00}$ x 100 = (COV) 9.13%

Conclusions:

1. Compare the COV's determined for each category.

   The COV for the improved residential category is considerably higher than for the residential land category. Excellent uniformity is indicated for the residential land, but improved residential uniformity is poor.

2. How do the COV's compare with the COD's you calculated on Worksheet #3?

   Results for both categories are higher than corresponding COD's.

APPENDIX III e

PRD
Worksheet #5

The PRD compares a weighted mean to the simple mean determined for the ratios. Index numbers greater than 1.0 indicate a tendency to favor higher priced properties, valuing them proportionately lower (lower ratios) than lower priced properties.

Procedure:

Use the data from the Sales Data Record to complete the following steps for each category of property.

Step #	Description of Step	Result Imp. Resid.	Result Res. Land
1	Sum of Assessor's market values	429,010.00	63,981.00
2	Sum of sales prices	576,671.00	68,200.00
3	Step 1 result / Step 2 result x 100 = weighted Mean ratio (%)	74.39	93.81
4	Sum of Ratios (%)	984.00	752.00
5	Sum of Ratios / n = Mean ratio (%)	82.00	94.00
6	Mean Ratio / Wtd. mean Ratio = PRD	1.10	1.00

Conclusions:

Compare each PRD.

The improved residential category PRD indicates a tendency to over-assess lower priced properties. Results on the residential land category show no tendency to favor either higher or lower priced properties.

APPENDIX III f

MEASUREMENTS OF UNIFORMITY
Shortcut Procedures

1. Coefficient of Dispersion (Resid. Land example).

a.	Sum of the ratios below the median in your array	349
b.	Sum of the ratios above the median in your array	403
c.	Subtract the sum in (a) from the sum in (b)	54
d.	Divide step (c) result by n	6.75
e.	Divide step (d) result by median; multiply by 100	7.11

2. Coefficient of Variation (Residential Land)

A Sale	B Ratio	C $(Ratio)^2$
1	80.00%	6400
2	84.00%	7056
3	92.00%	8464
4	93.00%	8649
5	97.00%	9409
6	99.00%	9801
7	103.00%	10609
8	104.00%	10816
Total	752.00	71204

STEPS:

1. Square each ratio and show result in Column C.
2. Find the sum of Column B.
3. Find the sum of Column C. 71204
4. Square the sum of Column B and divide the result by n.

$$\frac{(\text{Sum of Col. B})^2}{n} = \frac{565504}{8} = 70688$$

5. Subtract Step 4 result from Step 3 result 516
6. Divide the result in Step 5 by n-1 73.7143
7. Take the square root of your Step 6 answer 8.59
8. Divide Step 2 result by n to find the mean 94.00
9. Divide the standard deviation (Step 7) by the mean (Step 8) and multiply the result by 100.

$$\frac{\text{Standard Deviation}}{\text{Mean}} \times 100$$

$$\frac{8.59}{94.00} \times 100 = 9.13 = \text{COV}$$

Appendix III g

Using the Mann-Whitney test to determine equity between two property groups.

Mean Sale Price: $102,263

SP<mean Ratio	Rank	SP>mean Ratio	Rank	Sale Price	Assessed Value	Ratio $(\overline{A/S})$	Rank
		0.5174	1	$290,130	$150,127	0.5174	1
0.5294	2			$33,425	$17,694	0.5294	2
		0.5649	3	$286,500	$161,851	0.5649	3
		0.5953	4	$106,808	$63,580	0.5953	4
0.6175	5			$63,674	$39,320	0.6175	5
		0.6229	6	$243,525	$151,692	0.6229	6
0.6497	7			$71,625	$46,537	0.6497	7
		0.6592	8	$273,294	$180,156	0.6592	8
0.6683	9			$47,718	$31,890	0.6683	9
0.6713	10			$39,764	$26,692	0.6713	10
0.6729	11			$91,948	$61,875	0.6729	11
		0.6776	12	$119,375	$80,888	0.6776	12
0.7069	13			$91,326	$64,554	0.7069	13
0.7077	14			$77,025	$54,510	0.7077	14
0.7749	15			$58,750	$45,523	0.7749	15
0.7767	16			$74,960	$58,224	0.7767	16
0.7777	17			$56,760	$44,142	0.7777	17
0.7787	18			$58,464	$45,523	0.7787	18
0.7900	19			$56,476	$44,618	0.7900	19
		0.7996	20	$114,314	$91,409	0.7996	20
0.8033	21			$73,788	$59,272	0.8033	21
		0.8053	22	$247,750	$199,525	0.8053	22
		0.8054	23	$177,845	$143,236	0.8054	23
		0.8217	24	$112,930	$92,793	0.8217	24
		0.8240	25	$104,597	$86,191	0.8240	25
0.8287	26			$57,744	$47,850	0.8287	26
0.8423	27			$92,430	$77,855	0.8423	27
		0.8567	28	$135,000	$115,660	0.8567	28
0.8801	29			$56,220	$49,478	0.8801	29
0.8949	30			$40,291	$36,057	0.8949	30
0.9800	31			$43,785	$42,908	0.9800	31
		1.0323	32	$106,177	$109,602	1.0323	32
1.0480	33			$23,425	$24,549	1.0480	33
1.2834	34			$35,945	$46,132	1.2834	34
1.5049	35			$15,405	$23,183	1.5049	35
Total:	**422**		**208**				
Number:	**22**		**13**				
Average:	**19.2**		**16.0**				

U = [(22*13)+[22*(22+1)]/2]-422 $\qquad U = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$

U = 117

Z = [(117-((22-13)/2)][sqrt((22*13)*(22+13+1))/12]

Z = -0.88763

$$z = \frac{U - (n_1 n_2)/2}{\sqrt{(n_1 n_2)(n_1 + n_2 + 1)/12}}$$

The difference is not significant; value related inequity cannot be proven.

Appendix IV a

Values of the Standard Normal Distribution Function

z

Values of the Standard Normal Distribution Function										
z	0	1	2	3	4	5	6	7	8	9
-3.	.0013	.0010	.0007	.0005	.0003	.0002	.0002	.0001	.0001	.0000
-2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0238	.0233
-1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0300	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0570	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
- .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
- .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
- .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
- .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
- .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
- .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
- .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
- .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
- .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
- .0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

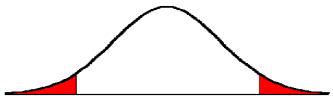
Appendix IV b

Values of the Standard Normal Distribution Function

Values of the Standard Normal Distribution Function										
Z	0	1	2	3	4	5	6	7	8	9
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	7995	.8023	.8051	.8078	.8106	.8133
9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9430	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9648	.9656	.9664	.9671	.9678	.9686	.9693	.9700	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9762	.9767
2.0	.9772	.9778	.9783	9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9874	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Appendix V

Table X Table of "Student's" Distribution: Value of t

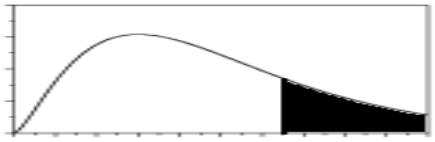


Degrees of Freedom	Probability												
	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01	0.001
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	636.619
2	0.142	0.289	0.445	0.617	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	31.598
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	12.924
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	8.610
5	0.132	0.267	0.408	0.559	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	6.869
6	0.131	0.265	0.404	0.553	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.959
7	0.130	0.263	0.402	0.549	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	5.408
8	0.130	0.262	0.399	0.546	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	5.041
9	0.129	0.261	0.398	0.543	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.781
10	0.129	0.260	0.397	0.542	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.587
11	0.129	0.260	0.396	0.540	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.437
12	0.128	0.259	0.395	0.539	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	4.318
13	0.128	0.259	0.394	0.538	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	4.221
14	0.128	0.258	0.393	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	4.140
15	0.128	0.258	0.393	0.536	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	4.073
16	0.128	0.258	0.392	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	4.015
17	0.128	0.257	0.392	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.965
18	0.127	0.257	0.392	0.534	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.922
19	0.127	0.257	0.391	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.883
20	0.127	0.257	0.391	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.850
21	0.127	0.257	0.391	0.532	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.819
22	0.127	0.256	0.390	0.532	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.792
23	0.127	0.256	0.390	0.532	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.767
24	0.127	0.256	0.390	0.531	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.745
25	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.725
26	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.707
27	0.127	0.256	0.389	0.531	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.690
28	0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.674
29	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.659
30	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.646
40	0.126	0.255	0.388	0.529	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.551
60	0.126	0.254	0.387	0.527	0.679	0.848	1.046	1.296	1.671	2.000	2.390	2.660	3.460
120	0.126	0.254	0.386	0.526	0.677	0.845	1.041	1.289	1.658	1.980	2.358	2.617	3.373
∞	0.126	0.253	0.385	0.524	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.291

This table is abridged from Table II of Fisher and Yates: Statistical Tables for Biological, Agricultural and Medical Research, published by Longman Group Ltd., London (previously published by Oliver & Boyd Ltd., Edinburgh) and by permission of the author and publishers.

Appendix VI

The Chi-Square Distribution



DF	Probability that Chi-Square value will be exceeded							
	.995	.990	.975	.950	.050	.025	.010	.005
1				.004	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	90.53	95.02	100.43	104.22
80	51.17	53.54	57.15	60.39	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	124.34	129.56	135.81	140.17

Appendix VII – Rules 130,131, and 315

130. DESCRIPTION OF PRIMARY CATEGORIES USED TO TEST FOR EQUALIZATION.
Sections 63-109 and 63-315, Idaho Code
Primary categories listed herein are for the purpose of testing values in each county and the Boise School District for equalization by the Tax Commission under Section 63-109, Idaho Code. (7-1-24)

01. Definitions. The following definitions apply for the purposes of testing for equalization under Section 63-109, Idaho Code, and reporting under Section 63-509, Idaho Code. (7-1-24)

a. Primary categories are used to study the following combinations of secondary categories: (7-1-24)

i. Vacant Residential Land: secondary categories 12, 15, 18 and 20; (7-1-24)

ii. Improved Residential Property: secondary categories 10, 12, 15, 18, 20, 26, 31, 34, 37, 40, 41, 46, 47, 48, and 50; (7-1-24)

iii. Vacant Commercial or Industrial Land: secondary categories 11, 13, 14, 16, 17, 21, and 22; (7-1-24)

iv. Improved Commercial or Industrial Property: secondary categories 11, 13, 14, 16, 17, 21, 22, 27, 33, 35, 36, 38, 39, 42, 43, and 51; (7-1-24)

v. Manufactured Housing: secondary categories 47 and 65; and (7-1-24)

vi. Agricultural Land: secondary categories 1 – 5. (7-1-24)

b. Secondary category means the categories established and described in Rules 510, 511, and 512 of these rules. (7-1-24)

c. See Conversion Table at https://tax.idaho.gov. (7-1-24)

02. Cross Reference. See Rules 509, 510, 511, and 512 of these rules. (7-1-24)

131. USE OF RATIO STUDY OR OTHER METHOD TO TEST FOR EQUALIZATION IN COUNTIES.
Section 63-109, Idaho Code

01. Equalization Ratio Study - Primary Categories Other than Agricultural Land. Each year the Tax Commission will conduct a ratio study to assist in the equalization of assessments of property within and among the primary categories, other than agricultural land, established in Rule 130 of these rules. The ratio study is conducted in accordance with the "Standard on Ratio Studies" and the "Standard on Verification and Adjustment of Sales" both referenced in Rule 003 of these rules. (7-1-24)

a. The annual ratio study will test assessments as of January 1 of each year. Assessments are tested using sales occurring between October 1 of the year preceding the year for which assessments are to be tested and September 30 of the year for which assessments are tested. Alternate time frames may be used when sales must be added to improve representativeness, or when an alternate study, as described in Subsection 09 of this rule, is to be used. Each sale price is adjusted for time and compared to market value for assessment purposes for the year for which assessments are to be tested. To improve representativeness, the Tax Commission may use sales from extended time periods, may add or delete sales, and may add appraisals when data is lacking. Equalization ratio studies must consist of at least five (5) sales and/or appraisals. Sales should be considered as potentially valid if a financial institution is the seller, provided that criteria found in the Standard on Verification and Adjustment of Sales are met. (7-1-24)

b. The study will be completed annually in March and notice provided to county official in accordance with Section 63-109, Idaho Code. For non-agricultural categories, the appropriate ratio study statistical measure of level is the median. For agricultural land categories, level of assessment is to be determined as described in Paragraph 131.02.b. of this rule. (7-1-24)

02. Equalization Study – Agricultural Land. Each year the Tax Commission will conduct a study to assist in the equalization of assessments of agricultural land. Any such study will analyze agricultural land values throughout each significant secondary agricultural land category using valuation methods found in Section 63-602K, Idaho Code, and Rule 617 of these rules. A secondary agricultural land category having at least ten percent (10%) of the acreage and at least five percent (5%) of the value of the primary agricultural land category is considered significant. (7-1-24)

a. County officials will receive notice of the results and compliance in accordance with Section 63-109, Idaho Code. (7-1-24)

b. Significant secondary agricultural land categories are subject to preliminary and follow-up studies of assessment level and are studied based on the valuation methodology described in Rule 617 of these rules. The preliminary study is a comparison to the prior year's assessed values. The follow-up studies will test the current year's assessed values and are required when preliminary studies indicate a level of assessment less than ninety percent (90%) or greater than one hundred ten percent (110%) of market value for assessment purposes. Categories meeting these criteria, and those categories not considered significant in a county, are in compliance. Level means the ratio of the median per acre assessed value and the median per acre value for the secondary agricultural land category determined by the Tax Commission using the valuation methodology found in Rule 617 of these rules. (7-1-24)

c. Secondary agricultural land categories may also be subject to follow-up studies if the Tax Commission has received information indicating that county boards of equalization have changed values in such a way as to produce likely non-compliance. (7-1-24)

03. Timing and Notification. Notice of improper assessment of any category is to occur when any category tested for equalization purposes is found out of compliance as described in this Rule. Following the first Monday in April statutory deadline for notice, additional notice will be provided as follows: (7-1-24)

a. By the second Monday in May, the Tax Commission will notify county assessors and commissioners of results of any additional ratio studies requested by county assessors. These studies will be based on current year assessments. (7-1-24)

b. By the fourth Monday in July, the Tax Commission will notify county assessors and commissioners of the results and compliance status based on follow-up studies as provided in Subsections 05 and 06 of this Rule. (7-1-24)

c. See Timing and Notification Table at https://tax.idaho.gov. (7-1-24)

04. Tested for Equalization. Except as provided in Subsection 131.05 of this rule, categories, other than agricultural land to be tested for equalization purposes, are the primary categories described in Subsection 130.01 of these rules. (7-1-24)

05. Follow-Up Ratio Study. If the annual ratio study indicates that assessments in any primary category are out of compliance with the standards of this rule, a follow-up study is required. In addition, if the Tax Commission is informed that a county board of equalization has implemented changes to assessments, likely resulting in a category failing compliance with the standards for the current year's assessments, a follow-up study is also required. A follow-up ratio study tests the assessments for January 1 of the year following the timeframe used in the preliminary agricultural study or the annual ratio study. The follow-up study uses property sales during the calendar year immediately preceding that date, unless use of an alternate time frame for sales will provide a more representative study. (7-1-24)

06. Notice of Follow-Up Ratio Study. The Tax Commission will notify the county commissioners, the county board of equalization, and the county assessor of the results of any follow-up study. The notification will include a description of assessment changes if such changes initiated the follow-up study. The notice will specify the compliance status of each category and will state whether and why the Tax Commission considers adjusting non-compliant categories based on the annual or follow-up ratio studies at the State Board of Equalization meeting. (7-1-24)

07. Use of Ratio Study Results. If the results of any ratio study show, with reasonable statistical certainty as defined in Subsection 131.11 of this rule, that the assessments are not equalized, the Tax Commission may, at its meeting pursuant to Section 63-108, Idaho Code, order the county auditor to adjust the value of property in the non-compliant category or categories or any portion of such category. Any adjustment factor recommended to the Tax Commission will be calculated by dividing the median level of assessment in the category or categories into one hundred percent (100%). Except as provided in Subsections 131.02 or 131.08 of this rule, adjustment will not be considered for any secondary category that does not have at least one (1) observation. (7-1-24)

08. Exception from Requirement for at Least One (1) Observation for Use of Secondary Category in Adjusted Value Determination. If the ratio study results warrant an adjustment to the assessed values of the primary residential category, secondary category 10 will receive a similar adjustment if at least one (1) property observation occurs in either category 12 or 15. Such adjustment is also warranted to the assessed values in secondary category 31 if at least one (1) observation occurs in secondary category 34 or 37. (7-1-24)

09. Use of Alternate Ratio Study. When the follow-up ratio study required by Subsection 131.05 of this rule does not measure the true assessment level, the Tax Commission may consider adjustments based on the most recent ratio study or other information relevant to equalization. (7-1-24)

10. Submission of Additional Information. Any party may request that the Tax Commission consider any information or studies relevant to equalization. Such a request will include a description of the information to be presented and conclusions drawn from the information. (7-1-24)

11. Reasonable Statistical Certainty. For the purposes of equalization of primary categories other than agricultural categories, "reasonable statistical certainty" that any primary category is not equalized is found if: (7-1-24)

a. The median ratio for the category(ies) being tested is less than ninety percent (90%) or greater than one hundred ten percent (110%) and a ninety percent (90%) two-tailed confidence interval around the median fails to include ninety percent (90%) or one hundred ten percent (110%); or (7-1-24)

b. An eighty percent (80%) two-tailed confidence interval around the median fails to include ninety percent (90%) or one hundred ten percent (110%) and this failure has continued for the current and most recent two year's ratio studies on the category(ies). (7-1-24)

12. Cross References. See Rules 130, 510, 511, 512, and 617. (7-1-24)

315. USE OF RATIO STUDY TO EQUALIZE BOISE SCHOOL DISTRICT (RULE 315).

Sections 63-315, 33-802(6), 50-2903, Idaho Code

01. Procedures for Boise School District Ratio Studies. The Boise School District ratio study is conducted in accordance with the "Standard on Ratio Studies" referenced in Rule 003 of these rules. (7-1-24)

a. Information on property sales, which meet the requirements of arm's length market value sales, is assembled into samples representing designations defined in Subsection 315.02 of this rule in the Boise School District. Except when sales or appraisals must be added or deleted to improve representativeness, sales used are those occurring within the Boise School District between October 1 of the year preceding the year for which adjusted market value is to be computed and September 30 of the next year. Each sale price is adjusted for time and compared to market value for assessment purposes for the year for which adjusted market value is to be computed. The Tax Commission may use sales from extended time periods and may add appraisals when data is lacking. (7-1-24)

b. The market value for assessment purposes of the sale or appraised property is divided by the adjusted sale price or appraised value to determine the ratio. (7-1-24)

c. A statistical analysis is conducted for the sales and appraisals in each property designation described in Subsection 315.02 of this rule in the Boise School District and appropriate measures of central tendency, uniformity, reliability, and normality computed. (7-1-24)

d. If fewer than five (5) sales and appraisals are available, no adjustment to the net taxable value of the designation is made. (7-1-24)

e. If it is determined with reasonable statistical certainty that the property designation is not at market value for assessment purposes, an adjusted market value is computed for the Boise School District by dividing the net taxable value for the year for which adjusted market value is to be determined by the appropriate ratio derived from the ratio study. The appropriate ratio to be used is the weighted mean ratio calculated from the sample for each designation, unless it can be clearly demonstrated that this statistic has been distorted by non-representative ratios. In this case the median may be substituted. (7-1-24)

f. Within the Boise School District, adjusted market value for each secondary category of real, personal and operating property is summed to produce the adjusted market value for the Boise School District. The Boise School District net taxable value is divided by this adjusted market value to produce the overall ratio of assessment in the Boise School District. (7-1-24)

g. Urban renewal increment values are not included in the net taxable value for the Boise School District. Upon receipt of an urban renewal agency's resolution recommending the adoption of an ordinance for termination of a revenue allocation area by December 31 of a given year, the increment value in the immediate prior year is included in the net taxable value for the Boise School District. If the resolution is received prior to the first Monday in April, the net taxable value for the immediate prior year is adjusted by adding the increment value. If any ratio study-based adjustments are warranted, they apply to the actual value including the increment value. If the resolution is received on or after the first Monday in April, but by September 1, a corrected certification of actual and adjusted values is provided as soon as practical. (7-1-24)

h. "Reasonable statistical certainty," that the property designation in question is not at market value for assessment purposes is tested using ninety percent (90%) confidence intervals about the weighted mean or median ratios. If the appropriate confidence interval includes ninety-five percent (95%) or one hundred five percent (105%), there is not "reasonable statistical certainty" that the property designation is not at market value for assessment purposes. (7-1-24)

i. Secondary categories are assigned to designations as follows: (7-1-24)

i. Secondary categories 10, 12, 15, 18, 20, 26, 31, 34, 37, 40, 41, 46, 47, 48, 65, or 50 are residential; and (7-1-24)

ii. Secondary categories 11, 13, 14, 16, 17, 21, 22, 27, 33, 35, 36, 38, 39, 42, 43, or 51 are commercial. (7-1-24)

j. For all secondary categories, described in Rule 510, 511, or 512 of these rules but not contained in the list in Paragraph 315.01.i. of this rule, adjusted market value will equal taxable value. (7-1-24)

k. "Appraisal" or "appraised value" refers to any Tax Commission provided property appraisal. (7-1-24)

02. Use of Property Designations. In computing the ratio for the Boise School District, the Tax Commission will designate property as residential or commercial and will assign sales and appraisals to these designations as shown in Paragraph 315.01.i. of this rule. For the Boise School District, adjusted market value is computed by dividing the appropriate ratio ascertained for each of these designations into the sum of the net taxable values for each secondary category assigned to a designation. Except as provided in Subsection 315.05 of this rule, for the net taxable value in any secondary category to be included in said sum, at least one (1) observation (sale or appraisal) from that secondary category must be present in the ratio study. If the ratio for any given designation in the Boise School District indicates that the market value for assessment purposes cannot be determined with reasonable statistical certainty to differ from statutorily required market value, the net taxable value shown on the Boise School District abstract(s) required pursuant to Subsection 315.04 of this rule for each of the secondary categories included in that designation is the adjusted market value for said designation. (7-1-24)

03. Assessor to Identify Location. Each county assessor will identify which sales submitted for the ratio study are located within the Boise School District. (7-1-24)

04. Abstracts of Value for the Boise School District. Each applicable county auditor will provide to the Tax Commission abstracts of the net taxable value of all property within the portion of the Boise School District in that county. These abstracts are submitted in the same manner and at the same time as provided for county abstracts of value. (7-1-24)

05. Exception from Requirement for at Least One Observation for Use of Secondary Category in Adjusted Value Determination. When there is an adjustment to be made to the net taxable values in the residential designation, such adjustment applies to any net taxable value in secondary category 10, provided there is at least one (1) observation (sale) of property identified in either secondary category 12 or 15. Such adjustment will also be applied to any net taxable value in secondary category 31, provided there is at least one (1) observation (sale) of property identified in either secondary category 34 or 37. (7-1-24)

06. Certification of Values. The Tax Commission certifies values under Section 63-315, Idaho Code, by publication on the Tax Commission's web site or in an alternate format on request. (7-1-24)

07. Cross References. See rules 130, 510, 511, and 512 of these rules. (7-1-24)